A practical assessment of risk-averse approaches in production lot-sizing problems

To appear in the *International Journal of Production Research*
Vol. 00, No. 00, 00 Month 20XX, 1–32

Revised manuscript: A practical assessment of risk-averse approaches in production lot-sizing problems

Douglas Alem[a*], Fabricio Oliveira[b], and Miguel Ruiz Peinado Carrión[c]

[a]*University of Edinburgh Business School, Management Science and Business Economics Group 29 Buccleuch Place, EH89JS, Edinburgh, UK*
[b]*Aalto University, Department of Mathematics and Systems Analysis P.O.Box 11100, 00076 AALTO, Finland*
[c]*Universidad de Castilla-La Mancha, Department of Electrical Engineering, Avenida Carlos III s/n, 45071 Toledo, Spain*

(*v5.0 released March 2018*)

This paper presents an empirical assessment of four state-of-the-art risk-averse approaches to deal with the capacitated lot-sizing problem under stochastic demand. We analyze two mean-risk models based on the semideviation and on the conditional value-at-risk risk measures, and alternate first and second-order stochastic dominance approaches. The extensive computational experiments based on different instances characteristics and on a case-study suggest that CVaR exhibits a good tradeoff between risk and performance, followed by the semideviation and first-order stochastic dominance approach. For all approaches, enforcing risk-aversion helps to reduce the cost standard deviation substantially, which is usually accomplished via increasing production rates. Overall, we can say that very risk-averse decision-makers would be willing to pay an increased price to have a much less risky solution given by CVaR. In less risk-averse settings, though, semideviation and first-order stochastic dominance can be appealing alternatives to provide significantly more stable production planning costs with a marginal increase of the expected costs.

Keywords: Lot-sizing; two-stage stochastic programming; risk-aversion; CVaR; semideviation; first-order stochastic dominance; second-order stochastic dominance.

1. Introduction

By recognizing that many practical production planning contexts are fraught with diverse sources of uncertainty, the literature on lot-sizing under uncertainty has been increasing over the last years. Non-deterministic lot-sizing problems are commonly addressed via probabilistic approaches, in which stochastic programming (SP) arises as the most popular (Aloulou et al. 2014). SP is based on the the knowledge of the probability distributions of uncertain parameters, which are usually represented by a finite set of realizations or scenarios. In fact, SP has the advantage of being an intuitive modeling approach to generate solutions that are able to hedge against multiple outcomes. Although stochastic programming is sometimes criticized for being computationally prohibitive for large-scale problems, Graves (2011) argued that with the increase in computational technology and the development of efficient algorithms, this technique has been more exploited for production planning problems in recent years.Stochastic lot-sizing formulations are mostly derived from either two-stage (Amorim et al. 2015; Hu and Hu 2016; Alem et al. 2018) or multistage structures (Huang and Küçükyavuz 2008; Li and Thorstenson 2014; Koca et al. 2018) in a *risk-neutral* (RN) perspec-

*Corresponding author. Email: douglas.alem@ed.ac.uk

tive, i.e., assuming that decision makers only focus on minimizing (maximizing) the expected cost (profit).

Despite the advantages of using stochastic programming models, the limitation of the traditional risk-neutral formulations in production planning and beyond is threefold. Firstly, RN models assume that the same decisions are taken under similar conditions repeatedly over a certain time period, implying by the Law of Large Numbers that, in the long-run, average and expected costs will coincide. But especially in a short-term period and depending on the degree of uncertainty, it is not possible to guarantee that the average of the first few results reflects a good decision along the planning horizon. Secondly, the expected value problem can generate production plans with good profits *on average*, but at the expense of experiencing very low profits, or even losses, in some unfavorable scenarios (Riis and Schultz 2003; Ruszczynski and Shapiro 2004; Shapiro et al. 2009). Thirdly, as the traditional stochastic programming leads to several and (possibly) very distinct scenarios, it is expected that the recourse decisions are too different as well. In practice, it means that production plans must be re-implemented every time a new scenario is materialized, which can be expensive and/or operationally infeasible (Vladimirou and Zenios 1997; Alem and Morabito 2013).

The aforementioned drawbacks can be overcome by including terms in the objective function that can measure exposition to risk and mitigate the effects of undesirable realizations, originating the so-called *risk-averse* approaches, which are commonly embedded into RN models. The idea of risk was first presented in Markovitz's pioneering paper on portfolio selection (Markowitz 1952, 1959), in which the author proposed measuring the risk of a portfolio by combining the expected return and the variance of the returns, defining a mean-risk model. In these models, risk is characterized as a scalar measure of the variability of outcomes. Markovitz inserted the variance in the objective function of the risk-neutral approach as a dispersion measure, considering that the larger the variance, the higher risk of experiencing a return different from the expected one. Therefore, the mean-risk model proposed by Markovitz allows to obtain solutions with a smaller variance at cost of decreasing the expected return. Although different authors studied the idea of mitigating the volatility of the random variables in a risk-averse perspective, Mulvey et al. (1995) was the first study that formalized a general approach to deal with robustness and risk reduction in scenario-based stochastic programs. Currently, the quantitative risk management addresses how to design representative and tractable *risk measures*, i.e., mathematical expressions to reflect the manager's preferences with respect to a set of random outcomes (Artzner et al. 1999; Ogryczak and Ruszczyński 1999, 2001; Takriti and Ahmed 2004; Schultz and Tiedemann 2006; Gollmer et al. 2008; Krokhmal et al. 2011; Alonso-Ayuso et al. 2014), as well as applying existing risk-averse models to different real-world applications, such as disaster management (Escudero et al. 2018; Alem et al. 2016), cash-flow (Righetto et al. 2019), structural topology optimization (Eigel et al. 2018), waste management (Toso and Alem 2014; Broitman et al. 2018), amongst many others.

Surprisingly, the aforementioned existing literature on stochastic lot-sizing problems almost always neglect the potential disadvantages of risk-neutral formulations. Therefore, references regarding risk-averse stochastic lot-sizing problems in the technical literature are scarce. The few exceptions we found did not elaborate on their choice for adopting a given risk-averse approach, nor compared them to other popular methodologies. In this line, Zhang et al. (2014) proposed joint chance constraints imposing an upper bound on the probability of a stockout within the whole planning horizon to deal with demand uncertainty in lot-sizing problems. This approach is rather different from using scenarios to describe the random nature of the uncertainty sources, and the goal is usually the fulfillment of given service levels. In order to assess the benefit of their new probabilistic formulation, they also developed a risk-averse lot-sizing model using the *mean-absolute deviation* (MAD) as risk measure. The results were not compared to other risk-averse approaches, but to robust optimization and pseudo-dynamic approaches. MAD has shown a poor performance in achieving good tradeoffs between expected costs and service levels. Within a scenario-based two-stage stochastic programming approach, Macedo et al. (2016) addressed a lot-sizing problem with remanufacturing option under several uncertain parameters in which risk was modelled using the *upper partial mean* through a mean-risk framework. The authors claimed that upper partial mean has the advantage of being asymmetric and tractable, while providing an intuitive risk analysis because this dispersion measure resembles the standard deviation. However, the fact that UPM often lead to suboptimal solutions to the second-stage problem problem in which variability is falsely reduced is well-known in the academic literature (Takriti and Ahmed 2004; Barbaro and Bagajewicz 2004). Mahmutogullari et al. (2018) presented a scenario tree decomposition approach to handle general risk-averse mixed-integer multi-stage stochastic problems based on the Conditional Value-at-Risk (CVaR) measure. Their approach is tested on a lot-sizing problem under stochastic costs. Despite the fact the authors present an interesting discussion on the effect of partition strategies and possible lower bound choices on the optimality gap of the algorithms, the performance of their approach concerning risk mitigation was not addressed.

Through the literature review and considering that the definition of risk is a subjective matter, we realized that, as there is no unrestrictedly recommendable risk-averse approach for production lot-sizing problems (or any other class of problems), the justification for adopting one risk-averse approach over another is usually given either regarding the preferences of the decision maker, the tractability of the resulting optimization model, or based on the properties of the risk measure. The authors of Artzner et al. (1999) proposed a set of desirable properties that risk measures should fulfill. These properties are i) translation invariance, ii) subadditivity, iii) positive homogeneity and iv) monotonicity. Measures satisfying these four properties are defined as *coherent risk measures.* Monotonicity implies that positions that lead to worse results present a greater risk. Translation Invariance guarantees that adding cash to a position reduces its risk by the same amount that it is added to it. Sub-additivity entails that the risk of a combined position is less than the sum of individual risks, which is known as the principle of risk reduction via diversification. The positive homogeneity axiom ensures that the risk proportionally increases or decreases with position size accordingly (Righi 2017; Krokhmal et al. 2011). Risk measures satisfying these properties then would behave appropriately, thus producing a suitable picture of risk exposure (Krokhmal et al. 2011).

In this context, the aim of this paper is to analyze four state-of-art risk-averse approaches to deal with risk mitigation in the capacitated two-stage stochastic lot-sizing problem with stochastic demands. Demand is deemed claimed to be one of the major exogenous uncertainty sources as it depends on market factors, seasonality effects, changing customer preferences, product life cycles, and so forth. We explore two different paradigms to handle risk based on mean-risk models and stochastic dominance constraints. In the first case, we include two different risk measures, the expected semideviation and the CVaR (Rockafellar and Uryasev 2000, 2002), in the objective function of the RN problem. These risk measures have been selected since they are coherent risk measures, as defined in Artzner et al. (1999), and can be mathematically formulated using linear expressions. If we consider a cost minimization problem, the expected semideviation can be defined as the difference between the expectation of those cost scenarios greater than the expected value and the expected cost. Observe that cost distributions with a low expected deviation are desirable to reduce the risk of experiencing costs greater than the expected one. A drawback of this risk measure is that it is not able to identify the fat tail corresponding to worst scenarios that may appear in cost distributions. However, the CVaR, for a given parameter α, is defined as the expectation of costs in $(1-\alpha)\cdot 100\%$ worst scenarios. Therefore, CVaR is able to detect the worst scenarios and optimize over them. On the other hand, we also analyze the usage of stochastic dominance constraints to handle the risk from a different point of view than typical mean-risk models, (Buckley 1986; Levy 1992; Dentcheva and Ruszczyński 2009). In this case, stochastic dominance allows to identify *acceptable* solutions and optimize over them. We assume that a solution is acceptable if the profit distribution associated with that solution *is better than* or *dominates* a pre-specified distribution that plays the role of *benchmark.* The preference of a profit distribution over another can be mathematically established using first- and second-order stochastic dominance constraints.

The major contribution of this paper lies on the development, evaluation, and systematic comparison of four risk-averse approaches for the capacitaded lot-sizing problem with stochastic demands within a scenario-based framework. For the first time, a novel formulation for the production lot-sizing problem with first- and second-order stochastic dominance constraints is presented. In particular, we devise a new way to model the first-order stochastic dominance principle, which is based on the ideas developed in Luedtke (2008). A systematic comparison of the four different risk hedging procedures against the corresponding risk-neutral solutions is vis-a-vis performed through several problem tests, thus constituting a contribution as well. We provided comprehensive numerical tests based on (i) 160 random generated instances based on the lot-sizing literature; and (ii) 10 larger instances inspired by a practical case-study in a soft-drink company. Interestingly, both sets of problems lead to rather similar implications regarding the risk performance of the four approaches. Our overall managerial insights suggest that all the methodologies hedge against uncertainty and risk similarly, but the tradeoffs may be quite different even for the same degree of protection. Moreover, we can have distinct choices for controlling the risk depending on how much the decision-maker accepts to pay to avoid potential losses. In particular, it is noteworthy that CVaR is the one that reduces most dramatically the cost in worst-case scenarios, but the corresponding price of risk-aversion is not necessarily negligible. On the other hand, semideviation and first-order stochastic dominance formulations are appealing to reduce the cost-standard deviation at a minor increase in the overall costs.

The remainder of the paper is structured as follows. Section 2 presents a two-stage stochastic version for the capacitated lot-sizing problem with lost sales under demand uncertainty. Section 3 develops the risk-averse extensions. Section 4 presents computational experiments and analyzes the risk-averse solutions. Concluding remarks and opportunities for future research are highlighted in Section 5.

2. Two-Stage CLSP with Lost Sales

We consider a two-stage version of the deterministic capacitated lot-sizing problem (CLSP) with lost sales and stochastic demands in which the main goal is deciding how producing $k = 1, \cdots, K$ products over $t = 1, \cdots, T$ time periods to fulfill stochastic demands at a minimum cost. Stochastic demand is represented by a finite number $\mathscr{W}$ of possible realizations on some probability space $(\Xi, \mathcal{F}, \mathbb{P})$, where Ξ is a set of possible states of nature with a generic realization denoted by ω equipped with a σ-algebra of events $\mathcal{F}$ with a probability measure $\mathbb{P}$ that belongs to a linear space $\mathcal{X}$ of $\mathcal{F}$-measurable functions $Z : \Xi \mapsto \mathbb{R}$. We say that π_ω is the probability associated with scenario ω, such that $\sum_\omega \pi_\omega = 1$ and $\pi_\omega > 0,\ \forall \omega$. The complete list of mathematical notation is given in the Appendix A.

According to the two-stage with recourse philosophy, the decision variables are partitioned into first-stage scenario-independent variables (production x_{kt}) and second-stage scenario-dependent decision variables (inventory $s_{kt\omega}$ and lost sales $\ell_{kt\omega}$). The unit production cost c_k, while inventory holding cost and lost sale penalty are given by h_k and p_k, respectively. Product k requires u_{kt} units of the total capacity cap_t to be manufactured. Finally, $d_{kt\omega}$ is the demand of product k in period t in scenario ω. The risk-neutral two-stage stochastic programming for the CLSP (RN) is posed as follows:

(RN)

minimize

$$\sum_{k=1}^{K}\sum_{t=1}^{T} c_k \cdot x_{kt} + \sum_{k=1}^{K}\sum_{t=1}^{T}\sum_{\omega=1}^{\mathscr{W}} \pi_\omega \cdot (h_k \cdot s_{kt\omega} + p_k \cdot \ell_{kt\omega}) \tag{1}$$

subject to:

$$x_{kt} + s_{k(t-1)\omega} - s_{kt\omega} + \ell_{kt\omega} = d_{kt\omega},\ \forall k = 1, \cdots, K \wedge t = 1, \cdots, T \wedge \omega = 1, \cdots, \mathscr{W} \tag{2}$$

$$\ell_{kt\omega} \leq d_{kt\omega},\ \forall k = 1, \cdots, K \wedge t = 1, \cdots, T \wedge \omega = 1, \cdots, \mathscr{W} \tag{3}$$

$$\sum_{k=1}^{K} u_{kt} \cdot x_{kt} \leq cap_t,\ \forall t = 1, \cdots, T \tag{4}$$

$$x_{kt},\ s_{kt\omega},\ \ell_{kt\omega} \geq 0,\ \forall k = 1, \cdots, K \wedge t = 1, \cdots, T \wedge \omega = 1, \cdots \mathscr{W}. \tag{5}$$

The objective function (1) consists of minimizing the expected total cost incurred in the first-stage production, and in the second-stage inventory and lost sales costs. The inventory balance constraints (2) define the stochastic constraint regarding demand, production, inventory, and lost sales for each product k, time period t, and scenario ω. Without loss of generality, we assume $s_{k0\omega} = 0$. Constraints (3) state upper bounds for the lost sales. The capacity constraints (4) take into account only production times. The remaining constraints (5) are the domain of the decisions variables.

Notice that the RN model is a special case of the classical (deterministic) multi-item capacitated lot-sizing problem with lost sales for $\mathscr{W} = 1$. From the stochastic programming viewpoint, CLSP-RN is a special case of full recourse − a so-called simple recourse − in which the recourse matrix, say W, is partitioned into identity matrices $W = [-\mathbf{I}, \mathbf{I}]$ associated with the recourse decisions $[\mathbf{s}, \boldsymbol{\ell}]$ simply evaluated according to the sign of $\mathbf{d}_\omega - \mathbf{x}$ for each scenario ω. Simple recourse formulations are especially appealing in production planning problems as we can ensure a feasible second-stage completion regardless the random variables realizations.

3. Risk-Averse Two-Stage CLSP with Lost Sales

In the following, we propose four alternate formulations to mitigate the risk associated with the variation of the recourse costs while capturing the decision maker's preferences towards risk. The first two models are built according to a mean-risk framework in which risk is either represented by the semideviation or the conditional value-at-risk measure. The last two models are based on first/second order stochastic dominance constraints.

3.1 *Mean-Risk Risk-Averse Models*

Mean-risk models are very popular in numerous real-world applications as they can conveniently compromise variability and cost satisfying risk-averse preferences. The "mean" term ($\mathbb{E}$) describes the expected outcome, whereas the risk term ($\mathbb{D}$) measures the variability of the outcome. Both objectives are thus combined as $(1 - \phi)\mathbb{E}[\mathbf{Z}] + \phi \cdot \mathbb{D}[\mathbf{Z}]$, where $\phi \in [0, 1]$ is the risk level. By parametrically varying ϕ, the efficient cost-risk frontier is generated and one can choose an optimum solution according to her/his risk attitude. We say that risk-aversion increases as ϕ increases.

Here, $\mathbb{D}[\mathbf{Z}]$ is either the semideviation or the CVaR risk measure. Both encompass dispersion and conditional expectation for the recourse cost, respectively. The motivation for adopting semideviation and CVaR as risk measures are their consistency with stochastic dominance principles − thus allowing us to search for stochastically non-dominated solutions (Ogryczak and Ruszczyński 2001) − and the computational tractability of the corresponding optimization problems.

The semideviation risk term $\mathbb{D}$ is defined as follows:

$$\mathbb{D} = \mathbb{E}[\max\{Z_{\mathbf{X}}(\omega), \mathbb{E}(Z_{\mathbf{X}}(\omega))\}], \tag{6}$$

in which $Z_{\mathbf{X}}(\omega) = \sum_{k=1}^{K}\sum_{t=1}^{T}(c_k \cdot x_{kt} + h_k \cdot s_{kt\omega} + p_k \cdot \ell_{kt\omega})$ is the (random) cost for scenario ω and $\mathbf{X} = [\mathbf{x}, \mathbf{s}, \boldsymbol{\ell}]$ is the decision vector. The corresponding semideviation-based mean-risk CLSP is written as:

(SD)

minimize

$$(1-\phi)\cdot\sum_{\omega=1}^{\mathscr{W}}\pi_\omega\cdot\lambda_\omega + \phi\cdot\sum_{\omega=1}^{\mathscr{W}}\pi_\omega\cdot\delta_\omega \tag{7}$$

subject to:

constraints (2) − (5)

$$\sum_{k=1}^{K}\sum_{t=1}^{T}(c_k \cdot x_{kt} + h_k \cdot s_{kt\omega} + p_k \cdot \ell_{kt\omega}) \le \lambda_\omega,\ \forall\omega = 1,\cdots,\mathscr{W} \tag{8}$$

$$\sum_{\omega'=1}^{\mathscr{W}}\pi_{\omega'}\lambda_{\omega'} \le \delta_\omega,\ \forall\omega = 1,\cdots,\mathscr{W} \tag{9}$$

$$\lambda_\omega \le \delta_\omega,\ \forall\omega = 1,\cdots,\mathscr{W} \tag{10}$$

$$\lambda_\omega,\ \delta_\omega \ge 0,\ \forall\omega = 1,\cdots,\mathscr{W}. \tag{11}$$

The objective function (7) is composed of the weighted combination between mean and risk terms. Constraints (8) capture the realized cost for scenario ω via the auxiliary variables λ_ω. Constraints (9) and (10) imply that the auxiliary variable δ_ω is defined as

$$\delta_\omega = \max\left\{\sum_{\omega}^{\mathscr{W}}\sum_{k=1}^{K}\sum_{t=1}^{T}\pi_\omega\cdot(c_k \cdot x_{kt} + h_k \cdot s_{kt\omega} + p_k \cdot \ell_{kt\omega}),\right.$$
$$\left.\sum_{k=1}^{K}\sum_{t=1}^{T}(c_k \cdot x_{kt} + h_k \cdot s_{kt\omega} + p_k \cdot \ell_{kt\omega})\right\}, \forall\omega = 1,\cdots,\mathscr{W}, \tag{12}$$

which becomes equivalent to (6) when considered as in the second term in (7). The risk term accumulates the average positive deviations for the second-stage cost (δ_ω), which is evaluated by $\sum_{\omega=1}^{\mathscr{W}}\pi_\omega\cdot\delta_\omega$. Constraints (11) state the domains of the new decision variables.

CVaR measures the expected cost exceeding a target η so-called Value-at-Risk (VaR) at the confidence level α, i.e., $\mathsf{CVaR}_\alpha = \mathbb{E}[Z_{\mathbf{X}}|Z_{\mathbf{X}} \ge \mathsf{VaR}_\alpha(Z_{\mathbf{X}})]$. More precisely, CVaR can be defined as

$$\mathsf{CVaR}_\alpha(Z) = \inf_{\eta\in\mathbb{R}} \left\{ \eta + \frac{1}{1-\alpha}\mathbb{E}\max[0, Z_{\mathbf{X}} - \eta] \right\}, \tag{13}$$

Within the mean-risk framework in which $\mathbb{D} = \mathsf{CVaR}_\alpha(Z_{\mathbf{X}})$, the corresponding optimization problem is posed as follows:

$$\begin{aligned}
&(\mathsf{CVaR})\\
&\text{minimize}\\
&(1-\phi)\left[\sum_{k=1}^{K}\sum_{t=1}^{T} c_k\cdot x_{kt} + \sum_{k=1}^{K}\sum_{t=1}^{T}\sum_{\omega=1}^{\mathscr{W}} \pi_\omega\cdot(h_k\cdot s_{kt\omega} + p_k\cdot \ell_{kt\omega})\right] +\\
&+\phi\cdot(\eta + \frac{1}{1-\alpha}\sum_{\omega=1}^{\mathscr{W}}\pi_\omega\vartheta_\omega) && (14)\\
&\text{subject to :}\\
&\text{constraints } (2)-(5)\\
&\vartheta_\omega \geq \sum_{k=1}^{K}\sum_{t=1}^{T}(c_k\cdot x_{kt} + h_k\cdot s_{kt\omega} + p_k\cdot \ell_{kt\omega}) - \eta,\ \forall\omega = 1,\cdots,\mathscr{W} && (15)\\
&\vartheta_\omega \geq 0,\ \forall\omega = 1,\cdots,\mathscr{W} && (16)\\
&\eta\in\mathbb{R}. && (17)
\end{aligned}$$

Constraints (15) and (17) imply that ϑ_ω is defined as

$$\vartheta_\omega = \max\left\{\sum_{k=1}^{K}\sum_{t=1}^{T}(c_k\cdot x_{kt} + h_k\cdot s_{kt\omega} + p_k\cdot \ell_{kt\omega}) - \eta, 0\right\},\ \forall\omega = 1,\cdots,\mathscr{W}, \tag{18}$$

that is, variable ϑ_ω is 0 if scenario ω has a cost lower than η. Otherwise, ϑ_ω assumes the difference between η and the corresponding cost, for all ω. Decision variables η and ϑ_ω can be interpreted as first- and second-stage decisions, respectively; see Schultz and Tiedemann (2006). The confidence level α serves to reflect risk preferences: larger values of α indicate more risk aversion, as the corresponding value-at-risk increases and CVaR controls more relative deviations.

3.2 *Stochastic Dominance Constraints*

The first- and second-order dominance models compare the random outcome with preselected benchmarks to determine "acceptable" solutions and optimize the resulting problem over them. An acceptable solution is feasible and dominates another one when it is "smaller" in some sense, assuming that smaller is better. Let Z_1 and Z_2 be two random outcomes or solutions. We say that Z_1 dominates Z_2 to first order , i.e. $Z_1 \preceq_1 Z_2$, iff $\mathbb{E}f(Z_1) \leq \mathbb{E}f(Z_2)$ for all nondecreasing functions f for which both expectations exist. In second order, $Z_1 \preceq_2 Z_2$, iff $\mathbb{E}f(Z_1) \leq \mathbb{E}f(Z_2)$ for all nondecreasing and convex functions f for which both expectations exist (Märkert 2004).

Equivalent conditions for first- and second-order can be devised by comparing the probability functions of the random outcomes. For first-order, $F_{Z_1}(\eta) = \mathbb{P}(Z_1 \leq \eta) \geq F_{Z_2}(\eta) = \mathbb{P}(Z_2 \leq \eta)$, or $\mathbb{P}(Z_1 \geq \eta) \leq \mathbb{P}(Z_2 \geq \eta)$ using the complement probability. The latter inequality clearly shows that

The risk-averse CLSP under first-order stochastic dominance constraints can be posed as follows:

(FOSD-O)

minimize

$$\sum_{k=1}^{K}\sum_{t=1}^{T} c_{kt} \cdot x_{kt} + \sum_{k=1}^{K}\sum_{t=1}^{T}\sum_{\omega=1}^{\mathscr{W}} \pi_{\omega} \cdot (h_k \cdot s_{kt\omega} + p_k \cdot \ell_{kt\omega}) + \sum_{p=1}^{\mathscr{P}} M \cdot \delta_p \tag{19}$$

subject to:

constraints (2) − (5)

$$\sum_{k=1}^{K}\sum_{t=1}^{T} (c_k \cdot x_{kt} + h_k \cdot s_{kt\omega} + p_k \cdot \ell_{kt\omega}) - \rho_p \leq M' \cdot \theta_{\omega p},\ \forall \omega = 1, \cdots, \mathscr{W} \wedge p = 1, \cdots, \mathscr{P} \tag{20}$$

$$\sum_{\omega=1}^{\mathscr{W}} \pi_{\omega} \cdot \theta_{\omega p} \leq \beta_p - \delta_p,\ \forall p = 1, \cdots, \mathscr{P} \tag{21}$$

$$\theta_{\omega p} \in \{0, 1\},\ \delta_p \in [0, 1],\ \forall \omega = 1, \cdots, \mathscr{W} \wedge p = 1, \cdots, \mathscr{P}, \tag{22}$$

in which M and M' are sufficiently large numbers. This formulation is the most often seen in the literature (see, for example Alonso-Ayuso et al. (2014)). In (20), the binary variables $\theta_{\omega p}$ serve as indicators (i.e., assume value $\theta_{\omega p} = 1$) whenever the cost associated with scenario ω is greater than the cost threshold ρ_p. The total probability of observing a scenario above a given cost threshold ρ_p is then calculated in (21) and compared against the cumulative probability β_p of benchmark point p, thus enforcing the first-order condition. One underlying premise of this formulation is that $\rho_p \leq \rho_{p+1}, \forall p = 1, \ldots, \mathscr{W} - 1$, which is trivially obtained by sorting the cost thresholds in ascending order. To guarantee a relatively-complete recourse, we included the slack variable δ_p that is penalized in the objective function by a sufficient large number M.

Preliminary experiments have shown that this formulation poses significant computational challenges, even for the small instances considered in this paper. Using this formulation, the solver was not able to return feasible solutions within 3600 seconds in nearly all cases tested. This computational setback has motivated us to consider an alternative formulation that is based on the developments presented in Luedtke (2008).

(FOSD)

minimize

$$\sum_{k=1}^{K}\sum_{t=1}^{T} c_{kt} \cdot x_{kt} + \sum_{k=1}^{K}\sum_{t=1}^{T}\sum_{\omega=1}^{\mathscr{W}} \pi_\omega \cdot (h_k \cdot s_{kt\omega} + p_k \cdot \ell_{kt\omega}) + \sum_{p=1}^{\mathscr{P}} M \cdot \delta_p \tag{23}$$

subject to:

constraints $(2)-(5)$

$$\sum_{k=1}^{K}\sum_{t=1}^{T} (c_k \cdot x_{kt} + h_k \cdot s_{kt\omega} + p_k \cdot \ell_{kt\omega}) \leq \sum_{p=1}^{\mathscr{P}} \rho_p \cdot \theta_{\omega p}, \ \forall \omega = 1, \cdots, \mathscr{W} \tag{24}$$

$$\sum_{p=1}^{\mathscr{P}} \theta_{\omega p} = 1, \ \forall \omega = 1, \cdots, \mathscr{W} \tag{25}$$

$$\varphi_p = \sum_{\omega=1}^{\mathscr{W}} \pi_\omega \cdot \theta_{\omega p}, \ \forall p = 1, \cdots, \mathscr{P} \tag{26}$$

$$\sum_{p'=1}^{p} \varphi_{p'} \geq \beta_p - \delta_p, \ \forall p = 1, \cdots, \mathscr{P} \tag{27}$$

$$\theta_{\omega p} \in \{0, 1\}, \ \varphi_p, \ \delta_p \in [0, 1], \ \forall \omega = 1, \cdots, \mathscr{W} \wedge p = 1, \cdots, \mathscr{P}. \tag{28}$$

In formulation (23)–(28), the auxiliary binary variables $\theta_{\omega p}$ are used to compute the cumulative probability of all scenarios w with a cost smaller than the benchmark point p by means of constraints $(24)-(26)$. The first-order dominance of the cost distribution over the benchmark is enforced by constraints (27). Notice that formulation (23)–(28) precludes the use of the parameter M' in constraints (24), which significantly improves the linear relaxations obtained in the solution process, thereby improving computational performance. An important difference between formulation (FOSD) and that presented in Luedtke (2008) is the presence of auxiliary variables φ_p whose role is solely to further strengthen the formulation. Notice that constraints (26) and (27) could be merged by substituting the former in the latter; however, we observed significant performance improvement by employing the auxiliary variables φ_p. The interested reader is referred to Luedtke (2008) for the theoretical background concerning the equivalence of both formulations.

For the second-order stochastic dominance, we also need to define the benchmark profile, which is now composed by (ρ_p, ϵ_p), for $p = 1, \cdots, \mathscr{P}$. In this case, ρ_p has the same meaning as in the FOSD (that is, target value or cost threshold) while ϵ_p represents the target for the expected cost shortfall (i.e., ϵ_p is the target for the expected value of those scenarios exceeding ρ_p). The CLSP under second-order stochastic dominance constraints reads:

(SOSD)

minimize

$$\sum_{k=1}^{K}\sum_{t=1}^{T} c_k \cdot x_{kt} + \sum_{k=1}^{K}\sum_{t=1}^{T}\sum_{\omega=1}^{\mathscr{W}} \pi_\omega \cdot (h_k \cdot s_{kt\omega} + p_k \cdot \ell_{kt\omega}) + \sum_{p=1}^{\mathscr{P}} M \cdot \delta'_p \tag{29}$$

subject to:

constraints (2) − (5)

$$\sum_{k=1}^{K}\sum_{t=1}^{T} (c_k \cdot x_{kt} + h_k \cdot s_{kt\omega} + p_k \cdot \ell_{kt\omega}) - \rho_p \leq \gamma_{\omega p},\ \forall \omega = 1, \cdots, \mathscr{W} \wedge p = 1, \cdots, \mathscr{P} \tag{30}$$

$$\sum_{\omega=1}^{\mathscr{W}} \pi_\omega \cdot \gamma_{\omega p} \leq \epsilon_p + \delta'_p,\ \forall p = 1, \cdots, \mathscr{P} \tag{31}$$

$$\gamma_{\omega p},\ \delta'_p \geq 0,\ \forall \omega = 1, \cdots, \mathscr{W} \wedge p = 1, \cdots, \mathscr{P}. \tag{32}$$

Constraint (30) defines the cost shortfall for each benchmark p and scenario ω, $\gamma_{\omega p}$, as the difference between the random cost $Z_{\mathbf{X}}(\omega)$ and the cost threshold ρ_p, i.e.,

$$\gamma_{\omega p} = \max\left\{\sum_{k=1}^{K}\sum_{t=1}^{T} (c_k \cdot x_{kt} + h_k \cdot s_{kt\omega} + p_k \cdot \ell_{kt\omega}) - \rho_p, 0\right\}, \forall \omega = 1, \cdots, \mathscr{W} \wedge p = 1, \cdots, \mathscr{P}, \tag{33}$$

while constraint (31) enforces that the second-order stochastic dominance holds. Once again, we included the slack variable δ'_p that is penalized in the objective function by a large number to guarantee relatively-complete recourse.

4. Computational Experience

Next, we discuss the computational experiments designed to analyze and compare the performance of the proposed models. We present a comprehensive analysis of the performance of all the risk-averse approaches in comparison to the standard risk-neutral model for several input data configurations and for two types of instances. The first set of 160 random generated instances are based on the lot-sizing literature. The second set of 10 instances are based on a real small-size soft-drink company. All the instances have been solved under five different settings (risk neutral plus four risk averse models) to allow for meaningful comparisons between the models. In total, 850 experiments have been performed with the aim to derive well-founded conclusions of the practical applicability of the proposed models.

Nevertheless, it is worth noting that the size and order of magnitude of the input data parameters of the test problems solved in this paper are similar to those encountered in other practical case studies consulted in the existing literature on lot-sizing problems; see, for example, Karagul et al. (2017); Güngör et al. (2018); Doostmohammadi and Akartunalı (2018). Moreover, the indicators used to assess the performance of the formulation, as described in Section 4.2, are also standard in the stochastic programming literature (see, for example, Alem and Morabito (2013)).

More specifically, we will attempt to answer the following questions:

Q1. How do the proposed approaches perceive a less risky solution?

Q2. How do the risk-averse approaches trade-off overall risk and performance?
Q3. What are the main advantages and disadvantages of each approach?
Q4. Are the risk-averse approaches sensitive to the characteristics of the problem?

All the models were coded in AIMMS 3.14 and solved using the optimization system ILOG-CPLEX 12.5 under default settings on an Intel i7 with 8GB RAM machine. A time limit of 3600 seconds and an optimality gap of 0.01% were set for all numerical experiments.

4.1 *Instance settings*

4.1.1 Overall data and parameters

Our test instances were generated according to existing literature on lot-sizing problems (Trigeiro et al. 1989; Maes et al. 1991; Zhou and Guan 2013; Wu et al. 2013). We built 16 classes of instances with 10 instances each, totaling 160 random test problems. The instances consider 7 or 15 periods, 10 or 20 products, time-wise dependent or time-wise independent demands with 100 equiprobable scenarios, and low or normal capacity utilization, in which the cumulative deterministic demand represents 85% (low) or 100% (normal) of the production capacity, respectively. The highlights of these instances are summarized in Table 1; notice that costs are per unit, periods are uniform-sized time intervals (e.g., days or weeks) and the capacity is measured in units of time (e.g. hours), while production times are considered per unit. The description and identification of each instance is detailed in Table 2.

Table 1.: Overview of the instance settings.

Parameter	Symbol	Levels
Number of periods	T	7 or 15
Number of products	K	10 or 20
Production cost	c_k	$U[10, 30]$
Inventory cost	h_k	$0.25 \cdot c_k$
Lost sales cost	p_k	$10 \cdot h_k$
Average deterministic demand	$\bar{d}_t$	$U[50, 100]$
Deterministic demand	d_{kt}	$U[\bar{d}_t - \frac{k}{100} \cdot \bar{d}_t, \bar{d}_t + \frac{k}{100} \cdot \bar{d}_t]$
Capacity	cap_t	$\frac{\sum_k d_{kt}}{0.85}$ or $\frac{\sum_k d_{kt}}{1.00}$
Production time	u_{kt}	$U[0.5, 1.5]$

Using the deterministic demands d_{kt} (which can be understood as an available forecast for the demand of product k for periods $t = 1, \ldots, T$) as a reference, the time-wise independent and dependent demands (referred hereinafter as independent or dependent) were obtained by means of Monte Carlo sampling. The samples were drawn from the stochastic processes $\tilde{d}_{kt}$ given by the expressions (34) and (35). In both cases, 100 samples $d_{kt\omega}$ (each sample containing $K \times T$ demand values) were drawn from $\tilde{d}_{kt}$.

The independent demand (ID) was assumed to follow

$$\tilde{d}_{kt} = \max\left\{N(d_{kt}, \frac{0.5 \cdot k}{K} \cdot d_{kt}), 0\right\}, \tag{34}$$

in which $N(\mu, \sigma)$ is the normal distribution with average μ and standard deviation σ. Notice that there is no dependence between time periods t, being the demand values only influenced by d_{kt}. Also, notice that products represented by higher value of k are subject to higher variability.

The dependent demand (DD) was modeled using

$$\tilde{d}_{kt} = \max\left\{\tilde{d}_{k(t-1)} + \varepsilon \cdot \frac{0.5 \cdot k}{K} \cdot d_{kt}, 0\right\}, \tag{35}$$

in which $\varepsilon \sim N(0,1)$ follows a normal distribution and $\tilde{d}_{k0} = d_{k1}$. In this, the process presents autocorrelation, since demands $\tilde{d}_{kt}$ depend on $\tilde{d}_{kt-1}$. Similarly to the independent case, products represented by higher value of k are subject to higher variability.

Figure 2 presents an example of demand scenarios generated for the 10th product (i.e., $k = 10$) ID and DD series, respectively.

Table 2.: Description of the proposed instances.

Instance	Products	Periods	Demand	Capacity	Identification[a]
1	10	7	Independent	Low utilization	1-p10-t7-c0.85-ID
2	10	14	Independent	Low utilization	2-p10-t14-c0.85-ID
3	10	7	Independent	Normal	3-p10-t7-c1-ID
4	10	14	Independent	Normal	4-p10-t14-c1-ID
5	20	7	Independent	Low utilization	5-p20-t7-c0.85-ID
6	20	14	Independent	Low utilization	6-p20-t14-c0.85-ID
7	20	7	Independent	Normal	7-p20-t7-c1-ID
8	20	14	Independent	Normal	8-p20-t14-c1-ID
9	10	7	Dependent	Low utilization	9-p10-t7-c0.85-DD
10	10	14	Dependent	Low utilization	10-p10-t14-c0.85-DD
11	10	7	Dependent	Normal	11-p10-t7-c1-DD
12	10	14	Dependent	Normal	12-p10-t14-c1-DD
13	20	7	Dependent	Low utilization	13-p20-t7-c0.85-DD
14	20	14	Dependent	Low utilization	14-p20-t14-c0.85-DD
15	20	7	Dependent	Normal	15-p20-t7-c1-DD
16	20	14	Dependent	Normal	16-p20-t14-c1-DD

[a] (Instance number)-k(# of products)-t(# of periods)-(capacity factor)-(demand type).

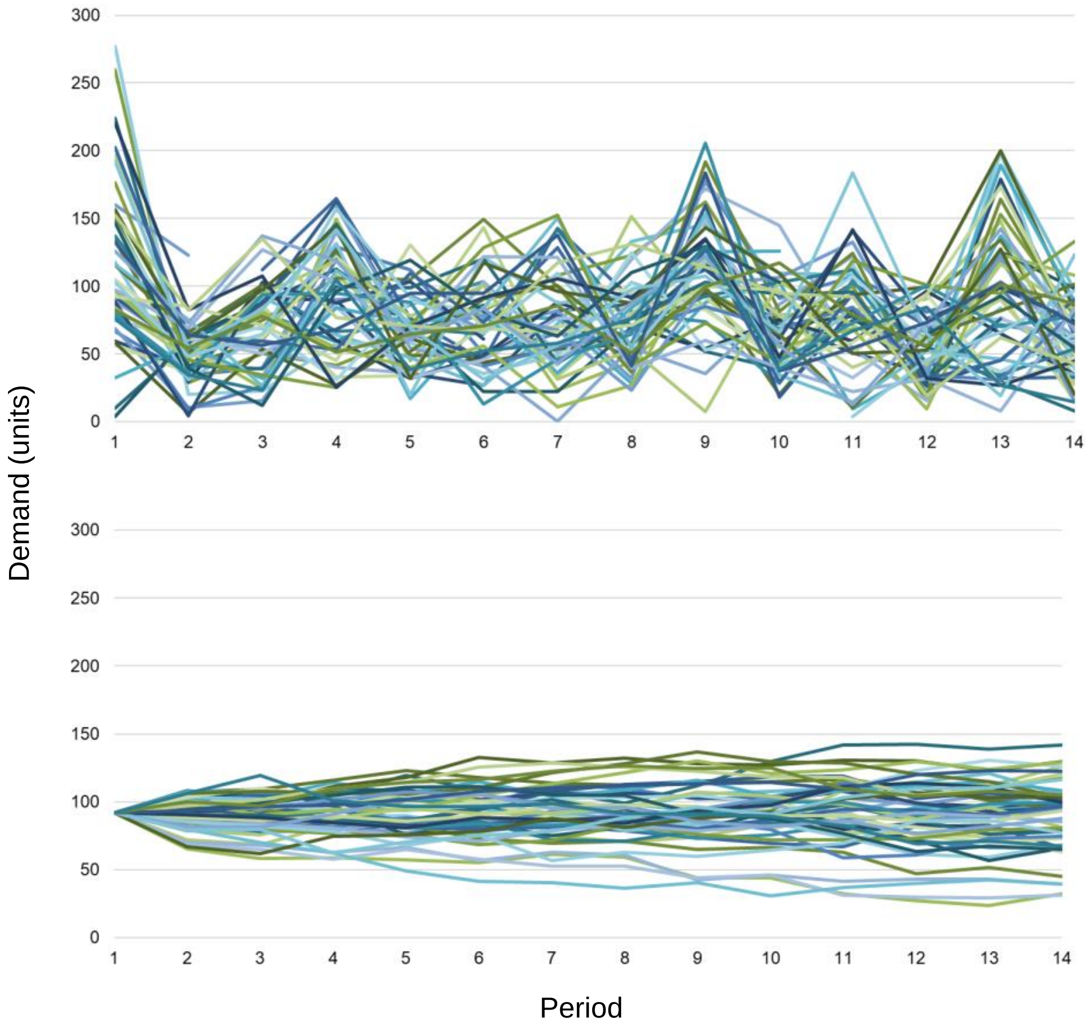


Figure 2.: Example of independent (above) and dependent (below) demand processes for product 20 in instances 8 and 16, respectively. The plots consider the first 50 scenarios only.

4.1.2 Risk-averse parameters

The parameters associated with each risk measure were set such that they could be meaningfully compared. For CVaR, we set $\alpha = 95\%$, which is usually the value used in the corresponding literature. Based on preliminary experiments in which we tried to balance the scale of the risk measures and the risk neutral parcels in the objective functions, we adopted $\phi = 0.99$ for both SD and CVaR. We stress that our focus is to enforce risk-averseness, which justifies our choice for the risk-averse parameter being in favor of risk-averse solutions. In spite of this rather overconservative ϕ value, our results clearly show that the price of risk-aversion is rather low and, on average, competitive with the stochastic dominance constraints that do not rely on a mean-risk objective function to limit the dispersion of the second-stage costs. Further analysis on distinct values of ϕ and the tradeoffs regarding expectation and risk for ϕ from 0.1 to 1 are discussed in the Appendix B.

The most challenging aspect associated with the use of FOSD and SOSD relates to the definition of the benchmark profiles to which the cost distribution should present the dominance relation. In the literature we could only find illustrative examples for toy cases, typically used for pedagogical purposes. In these examples, one can easily visualize the cost distribution and propose benchmark profiles, which is not the case for our study. Therefore, we decided to devise guidelines such that the benchmark profiles could be defined according to the information for the scenarios in each experiment and the solution obtained from the risk neutral model. For FOSD, we set two benchmark points. The first was obtained by calculating the 95% percentile of the cost distribution for the risk neutral model and reducing this value in 5% (ρ_1) and setting $\beta_1 = 0.95\%$. The second benchmark point was defined as the cost of the worst scenario increased by 50% (ρ_2), associated with cumulative probability of 100% (i.e., $\beta_2 = 100\%$). For the SOSD, we considered a single benchmark point, also set using the information obtained from the risk neutral model. In this case, we again used the 95% percentile of the cost distribution for the risk neutral model, reduced by 5% (i.e., ρ_1, as set for FOSD), and calculated the expected value of the 5% worst-case scenarios, which is also reduced in 5% (ϵ_1).

4.2 *Numerical Results and Discussion*

Figure 3 plots the average cost with the average values of the three performance indicators, i.e., standard deviation, worst-case, and CVaR at 95%. Costs are given in logarithmic scale to ease the visual representation. Table 3 presents the average results in terms of objective value ($\mathbb{E}$xpec), and related performance measures, such as standard deviation of the cost (StDev), worst-case scenario cost (Wc), and CVaR at 95% (CVaR$_{0.95}$). The risk-neutral results are given in absolute values for comparison purposes, e.g., for the first class of instances, expected cost of 261656 is given in monetary units, as well as the remaining performance measures. The risk-averse results are presented in terms of their relative difference (in percentage) with the risk-neutral values, with the exception of the elapsed times. Observations in Figure 3 and Table 3 are summarized as follows.

In general, all the remaining risk-averse models clearly outperform the risk-neutral in mitigating either the dispersion of the random cost or the right tail of the costs distribution given by the worst-case or CVaR$_{0.95}$ scenario. As expected, this improvement is achieved by means of a deterioration in the expected cost and it often leads to an increase computational burden. The tradeoff curves in Figure 3 indeed highlight that great risk mitigations might deteriorate up to 10% overall costs. Notice that the bottom left corner is the best quadrant, as it has minimum cost and maximum efficacy for all performance indicators, but the results are mainly concentrated in the quadrants to the right.

In particular, CVaR seems to dominate the other risk-averse approaches regarding the proposed performance indicators, except for the "price" of risk-aversion. The cost standard deviation was reduced 29.30% on average, at least 17.21% in instance `1-p10-t7-c0.85-ID`, and up to 52.43% in instance `11-p10-t7-c1-DD`. Surprisingly, such remarkable reductions were not followed by a similar increase in the expected cost. For example, in the most pessimistic situation given by instance `1-p10-t7-c0.85-ID`, we found that decision-maker would be willing to pay an additional cost of 6.69% to have a much less risky production plan in terms of standard deviation (−17.21%), worst-case cost (−4.32%) and CVaR$_{0.95}$ (−0.86%). In some instances, though, good tradeoffs between risk and performance are possible with increases of the expected costs smaller than 2%.

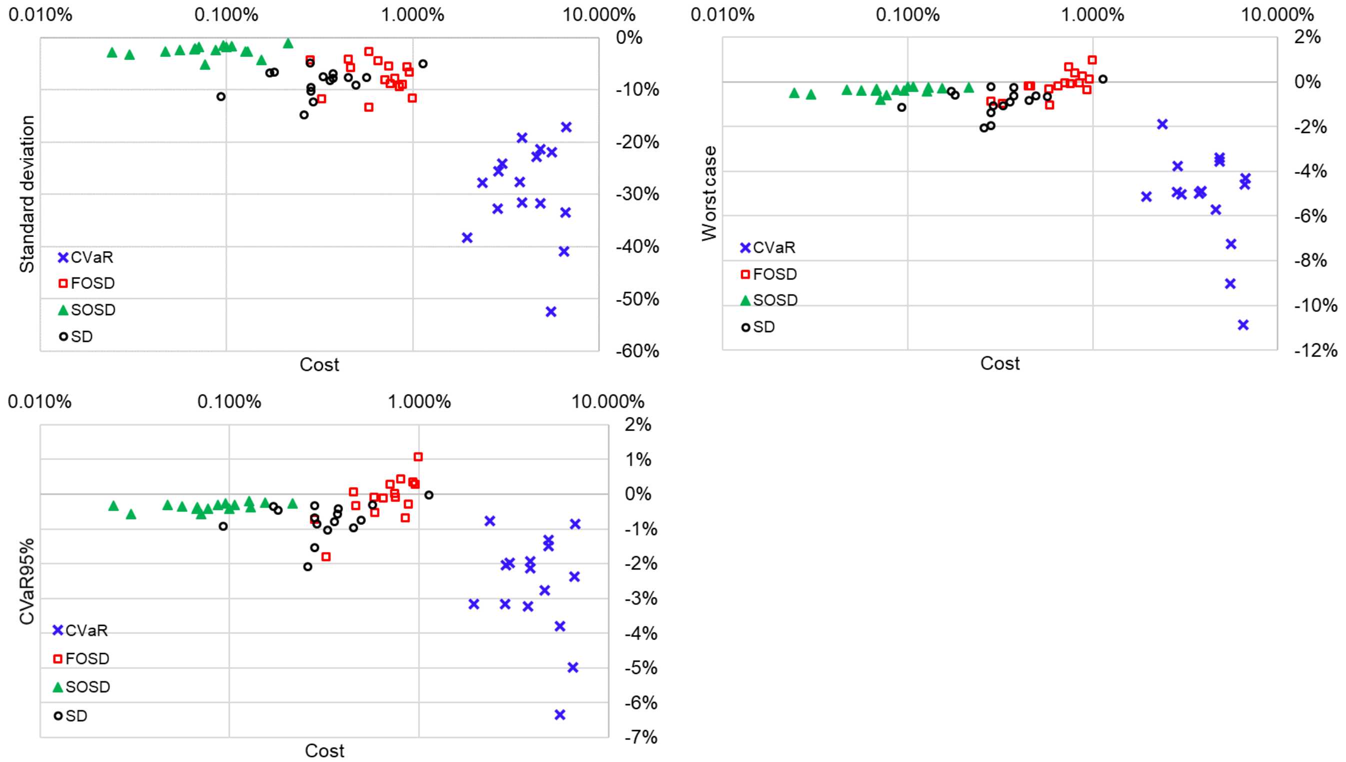


Figure 3.: Tradeoff curves between expected costs and the three performance indicators for all class of instances considered in this paper.

Table 3.: Summary of results for the proposed classes of instances and approaches in terms of expected value ($\mathbb{E}$xpec), standard deviation (StDev), worst-case scenario (Wc), and $\text{CVaR}_{0.95}$. All values refer to the average over ten instances.

	Benchmark (RN)	CVaR (%)	FOSD (%)	SOSD (%)	SD (%)	Benchmark (RN)	CVaR (%)	FOSD (%)	SOSD (%)	SD (%)
	1-p10-t7-c0.85-ID					5-p20-t7-c0.85-ID				
$\mathbb{E}$xpec	261656	6.686	0.583	0.214	1.137	457457	3.851	0.959	0.106	0.283
StDev	29158	−17.211	−2.756	−1.137	−5.107	36513	−19.207	−6.641	−1.656	−4.955
Wc	347364	−4.318	−0.323	−0.262	0.107	570754	−4.877	0.098	−0.220	−0.219
$\text{CVaR}_{0.95}$	326929	−0.863	−0.106	−0.263	−0.035	539319	−1.928	0.266	−0.300	−0.350
	2-p10-t14-c0.85-ID					6-p20-t14-c0.85-ID				
$\mathbb{E}$xpec	402212	4.626	0.744	0.100	0.374	910224	3.011	0.806	0.056	0.376
StDev	33724	−22.814	−5.558	−1.889	−6.989	50542	−24.121	−7.835	−2.475	−7.884
Wc	505754	−5.724	0.640	−0.232	−0.264	1065654	−5.046	0.375	−0.386	−0.647
$\text{CVaR}_{0.95}$	481024	−2.769	0.014	−0.423	−0.573	1021469	−1.982	0.439	−0.341	−0.423
	3-p10-t7-c1-ID					7-p20-t7-c1-ID				
$\mathbb{E}$xpec	179270	5.568	0.282	0.071	0.330	362690	4.839	0.451	0.095	0.569
StDev	21924	−21.984	−4.335	−1.773	−7.646	26215	−21.348	−4.222	−1.507	−7.751
Wc	244949	−7.246	−0.873	−0.804	−1.090	440133	−3.558	−0.187	−0.385	−0.679
$\text{CVaR}_{0.95}$	231297	−3.797	−0.741	−0.565	−1.051	422965	−1.321	0.058	−0.262	−0.311
	4-p10-t14-c1-ID					8-p20-t14-c1-ID				
$\mathbb{E}$xpec	324640	3.749	0.464	0.068	0.497	750043	2.847	0.997	0.024	0.292
StDev	22714	−27.620	−5.828	−2.101	−9.155	42288	−32.752	−11.703	−2.775	−12.329
Wc	394244	−5.001	−0.203	−0.326	−0.629	870314	−4.931	0.944	−0.497	−1.102
$\text{CVaR}_{0.95}$	378701	−3.228	−0.342	−0.383	−0.761	845156	−3.154	1.072	−0.335	−0.860
	9-p10-t7-c0.85-DD					13-p20-t7-c0.85-DD				
$\mathbb{E}$xpec	251835	4.838	0.8801	0.0869	0.3589	504780	2.373	0.7075	0.1535	0.1719
StDev	18166	−31.68	−9.010	−2.355	−8.305	21338	−27.73	−8.123	−4.306	−6.870
Wc	308635	−3.395	0.2600	−0.3631	−0.9146	565068	−1.889	−0.0757	−0.2946	−0.4461
$\text{CVaR}_{0.95}$	296349	−1.495	−0.2949	−0.2971	−0.8068	552525	−0.7662	0.2715	−0.2374	−0.3655
	10-p10-t14-c0.85-DD					14-p20-t14-c0.85-DD				
$\mathbb{E}$xpec	464938	6.620	0.8531	0.1297	0.4529	966932	2.883	0.7565	0.0469	0.1812
StDev	43074	−33.54	−9.420	−2.635	−7.742	56341	−25.63	−8.893	−2.722	−6.662
Wc	616249	−4.581	−0.0637	−0.2575	−0.8311	1129253	−3.754	−0.1108	−0.3447	−0.6197
$\text{CVaR}_{0.95}$	574219	−2.371	−0.6824	−0.3622	−0.9725	1095644	−2.045	−0.0893	−0.2990	−0.4657
	11-p10-t7-c1-DD					15-p20-t7-c1-DD				

Despite the fact that there is not a clear winner amongst the remaining approaches, both SD and FOSD generate good and similar tradeoffs between risk and performance, as we can confirm in the overlapped solutions depicted in Figure 3. In fact, SD (FOSD) achieves reductions up to 14.83% (13.40%) concerning the standard deviation with a corresponding increase of 0.26% (0.58%) in the objective function. In general, FOSD presents slightly higher expected costs than SD for similar levels of protection; e.g., instance `6-p20-t14-c0.85-ID`. The plots also evidence that SD solutions are more to the bottom/left than FOSD' solutions, indicating the superiority of the former in compromising costs and risk, which is expected due to its insensitiveness towards the actual magnitude of values above the set threshold. On the other hand, FOSD only captures whether costs are above the set threshold, but not by how much they exceed the threshold. However, notice that FOSD can be more effective than SD to reduce standard deviation in some class of instances, as in class `15-p20-t7-c1-DD`. All in all, risk-averse decision-makers who do not want to invest much in risk management strategies in exchange for a more reliable solution might certainly adopt the solution provided by either these two approaches.

SOSD was the least effective risk-averse approach, with a less prominent performance when compared to the others in terms of reducing the standard deviation. However, this measure presented often better performance in terms or reducing the magnitude of the worst-case and/or $\mathsf{CVaR}_{0.95}$ when compared with FOSD. In some cases, we observe that FOSD worsens the worst-case values because of the sub-optimality of some solutions, which in turn, increase the slack variable η_p. In fact, $\eta_p > 0$ means that the benchmark was modified to accommodate more scenarios above the 95% percentile (benchmark $\rho = 1$) or the worst-case benchmark (benchmark $\rho = 2$, which is equivalent to a 100% percentile). In these situations, no improvement at all it is expected. In Table 4, we present statistics of the slack variable and optimality gaps values observed for FOSD. For SOSD, all slack variables where zero at the optimum.

Table 4.: Average optimality gaps and slacks for FOSD.

	Average gap (%)	Average slack[a] (%)	# Non-optimal
`1-p10-t7-c0.85-ID`	0.00	0.00	–
`2-p10-t14-c0.85-ID`	0.00	0.00	–
`3-p10-t7-c1-ID`	0.00	0.00	–
`4-p10-t14-c1-ID`	0.00	0.00	–
`5-p20-t7-c0.85-ID`	2.00	0.14	3
`6-p20-t14-c0.85-ID`	6.40	3.56	5
`7-p20-t7-c1-ID`	1.00	0.00	1
`8-p20-t14-c1-ID`	4.00	0.62	1
`9-p10-t7-c0.85-DD`	4.00	0.00	3
`10-p10-t14-c0.85-DD`	3.50	0.00	2
`11-p10-t7-c1-DD`	5.00	0.00	1
`12-p10-t14-c1-DD`	1.00	0.00	1
`13-p20-t7-c0.85-DD`	9.00	3.83	9
`14-p20-t14-c0.85-DD`	6.75	1.81	4
`15-p20-t7-c1-DD`	8.50	2.42	6
`16-p20-t14-c1-DD`	7.00	2.78	4

[a] Average total reduction in the probabilities $(1 - \beta_\rho)$, for $\rho = 1, 2$.

Figure 4 plots the cost distribution curve for `3-p10-t7-c1-ID` (tenth run) as an illustrative example of the effect of each of the risk measures. The x-axis presents the scenarios and the y-axis present the cost observed for each scenario. The plots present the costs ordered by magnitude, showing an approximation for their cumulative distribution. Notice that all the scenarios are equiprobable with probability $\frac{1}{|\mathscr{W}|}$. This figure illustrates some of the effects previously discussed, in particular, the increase of the expected cost caused by CVaR and the prominent worst-case costs observed for FOSD.

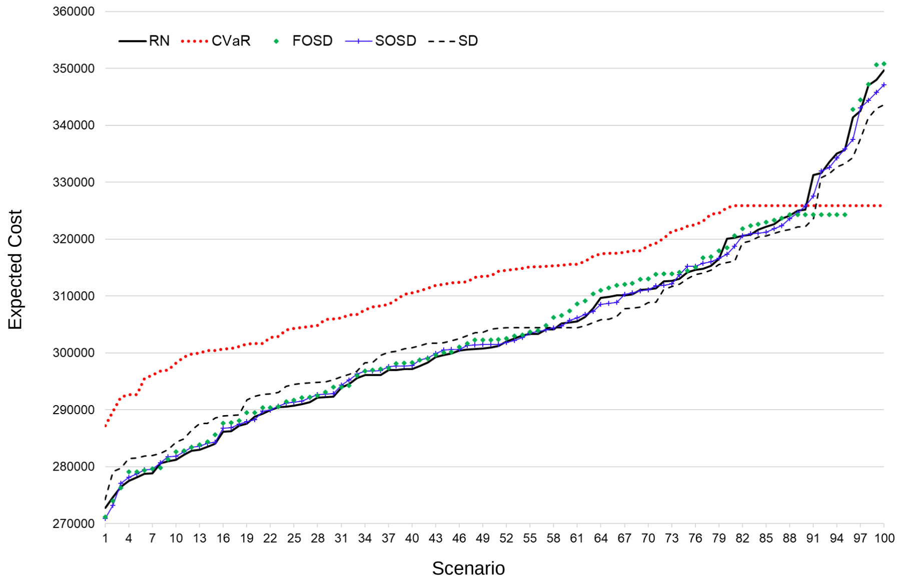
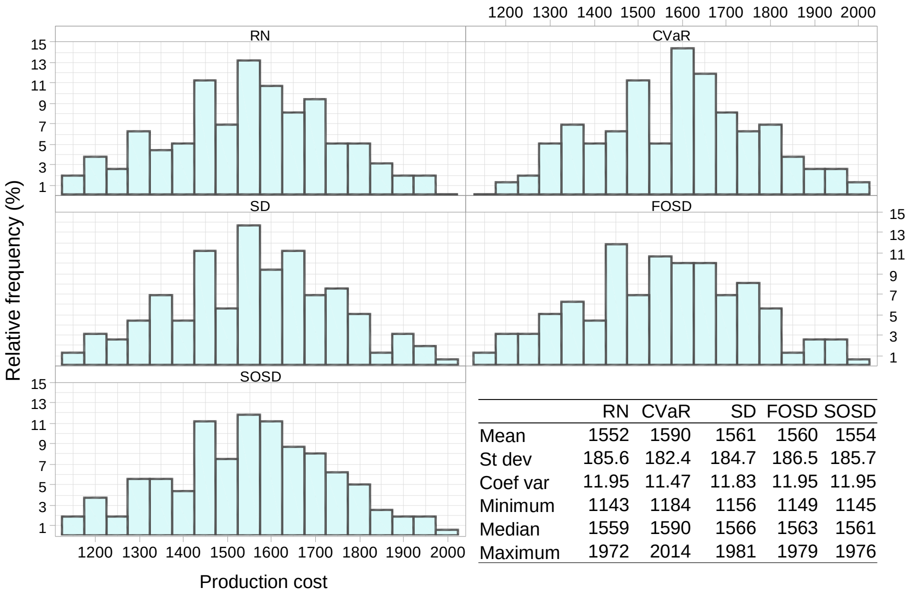


Figure 4.: Cost distribution for `4-p10-t7-c1-ID` (run number 10).

Finally, Figure 5 depicts the histograms of the first-stage production costs over the sample composed of 10 runs for each class of instance (totaling 160 costs) for all approaches. The relative frequency refers to the number of times that each cost value is observed within the sample. The descriptive statistics also show the mean value; the standard deviation (St dev); the coefficient of variation, evaluated as (St dev/mean)%, the minimum production cost, the median, and the maximum production cost. The behavior of the costs suggests that production lot-sizing policies are only marginally affected by enforcing risk-aversion via stochastic dominance constraints. On the other hand, the mean-risk models exhibit the highest production rates, suggesting that those approaches achieve substantial risks reductions by increasing production, which may increase inventory costs, but decrease lost sales. Clearly, this behavior is more pronounced for CVaR whose probability of yielding a production cost greater than the average value 1552, for example, is 13.1%, against up to 3.6% for the remaining approaches.



	RN	CVaR	SD	FOSD	SOSD
Mean	1552	1590	1561	1560	1554
St dev	185.6	182.4	184.7	186.5	185.7
Coef var	11.95	11.47	11.83	11.95	11.95
Minimum	1143	1184	1156	1149	1145
Median	1559	1590	1566	1563	1561
Maximum	1972	2014	1981	1979	1976

Figure 5.: Histograms of the first-stage production costs over the sample composed of 10 runs for each class of instance (totaling 160 costs) for all approaches.

Table 5 shows the average values for eight classes of instances, named: independent or dependent demands; 10 or 20 products; 7 or 14 time periods; tight or normal capacity. It is clear that most risk-averse models have achieved better risk reductions for dependent demand and normal capacity instances. This result was particularly evident in CVaR, whose risk reductions are up to 32% better under normal capacity, which is expected given that less risk solutions present higher production rates in general. Interestingly, it seems that FOSD is more effective than SD to reduce the standard deviation under tighter capacity instances. There is not a clear trend between the number of products/periods and risk mitigation strategies. Apparently, though, mean-risk approaches are more effective in instances with 10 products, while the remaining models are slightly better to reduce overall risk under 20 products.

Table 5.: Average values over the 8 classes of independent (ID) or dependent demands (DD); 10 (p10) or 20 (p10) products; 7 (t7) or 14 (t14) time periods; tight (c0.85) or normal (c1) capacity in terms of expected value ($\mathbb{E}$xpec), standard deviation (StDev), worst-case scenario (Wc), and $\mathsf{CVaR}_{0.95}$. All values are given in percentage with reference to the RN results

	CVaR				SD			
	$\mathbb{E}$xpec	StDev	Wc	$\mathsf{CVaR}_{0.95}$	$\mathbb{E}$xpec	StDev	Wc	$\mathsf{CVaR}_{0.95}$
DD	4.317	−35.21	−5.444	−2.911	0.2608	−9.454	−1.178	−0.9835
ID	4.397	−23.38	−5.088	−2.380	0.4822	−7.727	−0.5653	−0.5455
p10	5.511	−31.03	−6.272	−3.230	0.4618	−8.761	−0.9616	−0.9793
p20	3.203	−27.57	−4.259	−2.062	0.2811	−8.420	−0.7819	−0.5497
t7	4.454	−28.73	−4.933	−2.460	0.4004	−8.348	−0.8115	−0.7426
t14	4.260	−29.87	−5.599	−2.832	0.3425	−8.833	−0.9320	−0.7864
c0.85	4.361	−25.24	−4.198	−1.777	0.4169	−6.814	−0.4793	−0.4990
c1	4.353	−33.35	−6.333	−3.514	0.3260	−10.37	−1.264	−1.030
Average	4.357	−29.30	−5.266	−2.646	0.3715	−8.590	−0.8718	−0.7645
	FOSD				SOSD			
	$\mathbb{E}$xpec	StDev	Wc	$\mathsf{CVaR}_{0.95}$	$\mathbb{E}$xpec	StDev	Wc	$\mathsf{CVaR}_{0.95}$
DD	0.7114	−8.858	−0.3266	−0.3668	0.0896	−3.179	−0.3980	−0.3485
ID	0.6608	−6.110	0.0589	0.0825	0.0919	−1.914	−0.3891	−0.3590
p10	0.5978	−6.659	−0.2193	−0.5119	0.0959	−2.176	−0.3987	−0.4099
p20	0.7744	−8.309	−0.0483	0.2275	0.0857	−2.917	−0.3884	−0.2976
t7	0.5966	−7.544	−0.3945	−0.3612	0.1043	−2.660	−0.4339	−0.3620
t14	0.7756	−7.424	0.1269	0.0769	0.0772	−2.434	−0.3532	−0.3455
c0.85	0.7862	−7.279	0.1012	−0.0227	0.1117	−2.397	−0.2951	−0.3154
c1	0.5860	−7.689	−0.3689	−0.2616	0.0699	−2.696	−0.4920	−0.3922
Average	0.6861	−7.484	−0.1338	−0.1422	0.0908	−2.547	−0.3936	−0.3538

In terms of computational effort, Table 6 shows that CVaR and SD have similar elapsed times than those provided by the RN model, while SOSD presented a non-negligible increase, ranging from 2 to 17 times. While RN, CVaR, and SD are all linear programming problems, the FOSD model requires the inclusion of $\mathscr{W} \cdot \mathscr{P}$ integer variables, which makes the problems more challenging computationally. This becomes clear when we compare the average elapsed time for FOSD and the remaining models. Also, FOSD was the only approach that failed to provide the optimality certificate (which in MIP means reaching an optimality gap less or equal than the tolerance of 0.01%) within the time limit of 3,600 seconds, as depicted in Table 4. The characteristics of the proposed classes of instances also have interesting implications into the computational effort of the models. For example, the worse performances of the RN model are found for longer time horizons, but increasing the number of products causes the worst elapsed times for the risk-averse models, particularly for the FOSD model. One can notice that the FOSD strategy struggles more to solve demand-dependent instances, as these are typically more challenging due to larger cumulative demand in high demand levels in scenarios throughout the planning horizon, and thus presenting the highest costs for these worst-case scenarios.

4.3 *Larger Instances from a Soft-Drink Company*

In this section, we analyse a realistic case-study based on a real small-size company that produces carbonated soft-drinks in the countryside of São Paulo State, in Brazil. This company produces more than 22.5 millions of units per year. The objective of this case-study is to test the performance of the four risk averse approaches described in Section 3 in a real-life case-study.

Table 6.: Average elapsed times in seconds of the proposed approaches.

Class	RN	CVaR	FOSD	SOSD	SD
1-p10-t7-c0.85-ID	0.8063	0.8907	5.783	2.859	1.261
2-p10-t14-c0.85-ID	3.089	4.205	52.48	7.788	4.897
3-p10-t7-c1-ID	0.6297	0.7843	6.797	0.9499	0.9968
4-p10-t14-c1-ID	2.003	3.477	31.86	5.462	2.905
5-p20-t7-c0.85-ID	1.841	3.716	449.4	22.31	5.497
6-p20-t14-c0.85-ID	7.498	19.97	2030	30.25	24.01
7-p20-t7-c1-ID	1.302	2.955	114.7	19.64	5.076
8-p20-t14-c1-ID	5.567	17.12	1389	21.40	16.81
9-p10-t7-c0.85-DD	0.9796	0.7937	1.296	38.44	2.305
10-p10-t14-c0.85-DD	3.497	3.739	3.728	37.20	8.286
11-p10-t7-c1-DD	0.7268	0.6813	0.9375	25.31	4.188
12-p10-t14-c1-DD	2.617	2.809	4.438	20.03	13.15
13-p20-t7-c0.85-DD	2.741	3.9391	6.548	1547	17.63
14-p20-t14-c0.85-DD	8.956	13.26	29.36	1374	118.2
15-p20-t7-c1-DD	1.636	2.747	5.347	991.0	20.19
16-p20-t14-c1-DD	7.737	15.83	31.10	966.7	118.1
Average[a]	3.227	6.057	567.5	25.79	9.013
Average[b] ID	2.842	6.640	510.2	13.83	7.681
Average[b] DD	3.611	5.475	624.9	37.74	10.34
Average[b] p10	1.794	2.172	27.24	5.624	2.557
Average[b] p20	4.660	9.942	1107	45.95	15.47
Average[b] t7	1.333	2.063	397.3	11.26	3.370
Average[b] t14	5.121	10.05	737.7	40.32	14.66
Average[b] c0.85	3.676	6.314	691.7	26.197	9.575
Average[b] c1	2.777	5.801	443.3	25.379	8.451

[a] Average over the 16 classes of instances.
[b] Average over the 8 classes of independent (ID) or dependent demands (DD); 10 (p10) or 20 (p10) products; 7 (t7) or 14 (t14) time periods; tight (c0.85) or normal (c1) capacity.

4.3.1 Input data

We consider the 37 most important products (those with the largest cumulative demand) that this company weekly manufactures within a planning horizon of 6 months composed of 26 production weeks. The company has provided the observed demand for all the products over the considered planning horizon. Notice that a weekly time resolution has been adopted in this case-study, where each time period t refers to a single week composed of 5 working days. Figure 6 represents the annual demand of each considered product in descending order. The maximum and minimum annual demands in this set of products are 7.5 and 0.04 million units, respectively. As an example, Figure 7 provides the actual historical demands of four representative products during the 26-week planning horizon. Note that there exists a significant correlation between the demand of these four products.

Considering the absence of historical data concerning the realised demands, we followed our developed methodology to generate the scenarios based on the real deterministic demands. This way, the demand scenarios in period t are assumed to be governed by a uniform distribution with a mean equal to $\bar{d}_t$, $U[\bar{d}_t - \frac{x}{100} \cdot \bar{d}_t, \bar{d}_t + \frac{x}{100} \cdot \bar{d}_t]$, in which $\bar{d}_t$ is the historical demand in week t. By means of Monte Carlo sampling, ten different instances of 100 scenarios have been randomly generated. It is worth mentioning that the same procedure was used to generate the scenarios due to the absence of historical data. It is worth mentioning that overall costs are given in Brazilian Reais (BRL) per unit of product manufactured, per unit of product kept in inventory, and per unit of product that is not produced. The capacities are given in liters of soft-drink. These figures will be omitted for reasons of confidentiality.

Finally, the benchmark profiles used in this case study for FOSD and SOSD consist of two points that have been generated based on the cost distribution resulting from solving the risk-neutral problem, as explained in Section 4.1.2.

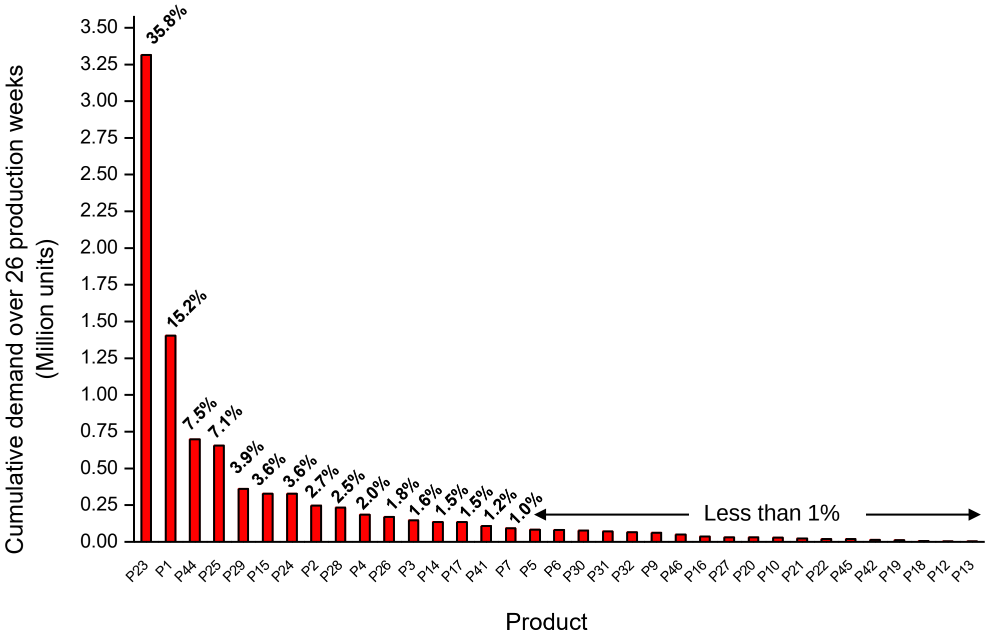


Figure 6.: Cumulative demand for all the 37 products over 26 production weeks and corresponding percentages.

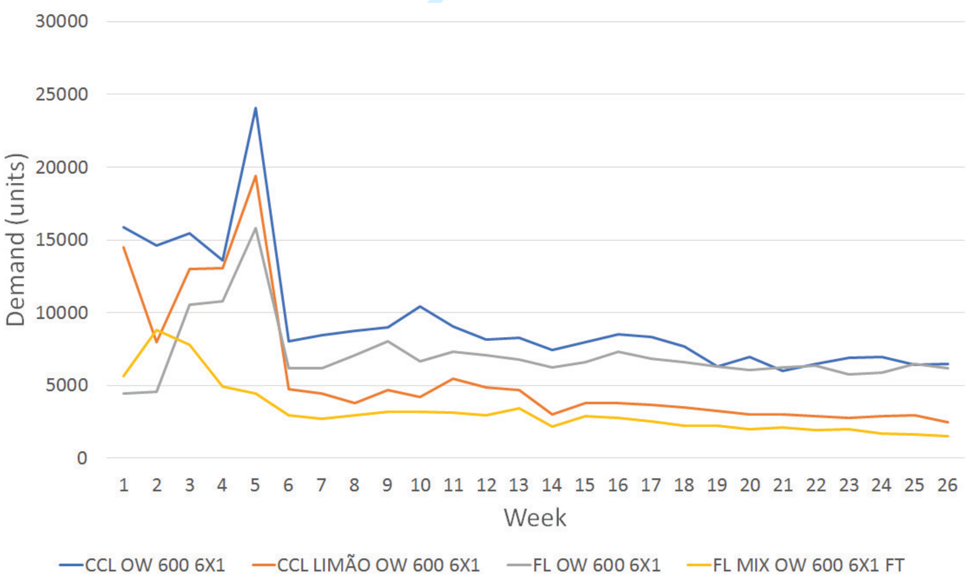


Figure 7.: Example of demand data for 4 products.

4.3.2 Results

Table 7 presents the average results in terms of objective value ($\mathbb{E}$xpec), and related performance measures, such as standard deviation of the cost (StDev), worst-case scenario cost (Wc), and

CVaR at 95% ($\mathsf{CVaR}_{0.95}$). Again, the risk-neutral results are given in absolute values for comparison purposes, e.g., the expected cost of 294,423,718 is given in monetary units, as well as the remaining performance measures. Table 7 and Figure 8 confirm that CVaR was the most efficient measure in terms of overall risk mitigation, at the expense of an average increase of 8.155% in expected costs. The other risk measures also presented a similar qualitative behaviour as the instances analysed before, with milder effects on the performance measures observed. For example, SD provided a reduction of 12.934% in StDev, which is higher than almost all classes considered in Section 4.2, with the exception of a 14.83% reduction for Class 11. It is also observed that FOSD and SOSD are less efficient than CVaR and SD to reduce the cost in terms of the standard deviation, worst-case cost and $\mathsf{CVaR}_{0.95}$. Nevertheless, less conservative managers might benefit from the stochastic dominance solutions. For example, implementing the FOSD risk-averse approach means having a 6.2% more assertive production plan at expenses of a negligible price. Moreover, the histograms reveal that the ranged spanned by the production cost given by the FOSD approach is narrower than in the risk-neutral model, indicating that the probability of more pessimistic (undesirable) scenarios in slightly lower in the former. These findings sound particularly relevant for small-size soft-drink companies that might not be able to invest much in their facilities, thus focusing on cheaper risk management actions can be an alternative to cope with demand fluctuation. These conclusions are similar to those that can be drawn from analyzing the random generated instances presented in Table 3. Further results obtained from solving this case-study are presented in Appendix C.

Table 7.: Summary of results for the larger instances and for all and approaches in terms of expected value ($\mathbb{E}$xpec), standard deviation (StDev), worst-case scenario (Wc), and $\mathsf{CVaR}_{0.95}$. All values refer to the average over ten instances.

	Benchmark (RN)	CVaR (%)	FOSD (%)	SOSD (%)	SD (%)
$\mathbb{E}$xpec	294,423,718	8.155	0.190	0.041	0.344
StDev	34,999,684	−44.623	−6.257	−2.689	−12.934
Wc	434,435,449	−9.570	−0.215	−0.433	−1.388
$\mathsf{CVaR}_{0.95}$	396,330,462	−8.190	−1.291	−0.653	−2.584

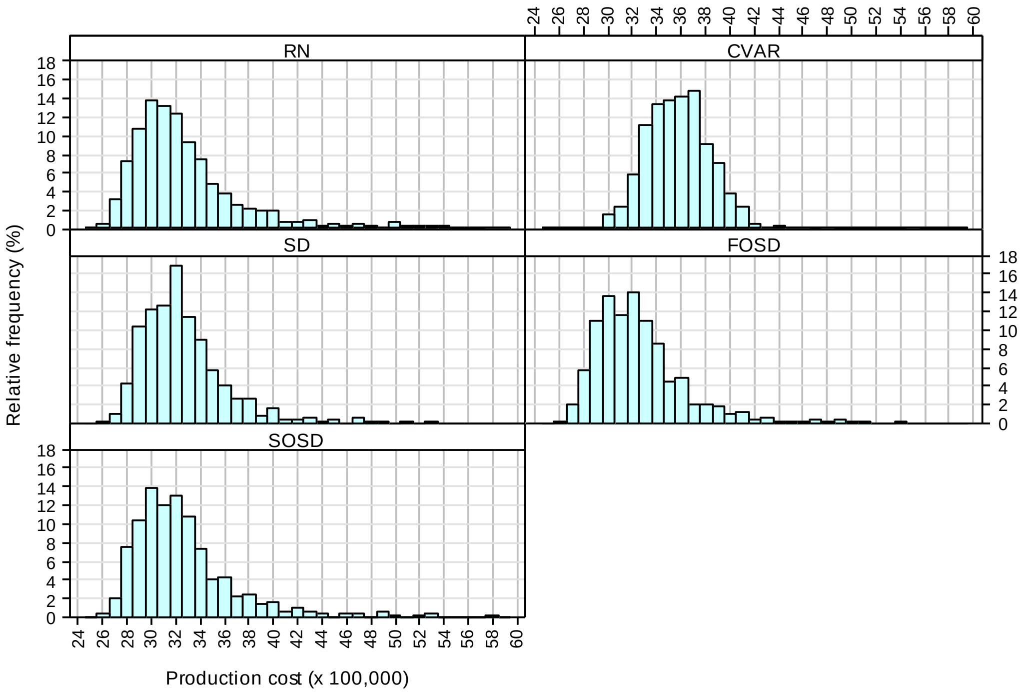


Figure 8.: Histograms of the expected total costs for the ten larger instances inspired by the case-study, which are composed of 100 scenarios each (totaling 1,000 costs) for all approaches.

4.4 *Managerial Implications*

All the analyzed approaches mitigates risk by reducing the standard deviation of the second-stage costs, whilst worst-case values were not much reduced in general. Thus, a less risky solution presents a less dispersed recourse cost. CVaR is the only approach that reduces fairly the right-tail of the recourse costs. In this sense, whether the dispersion of the outcomes is not critical, but avoiding overly expensive production costs is, the decision-maker should consider to increase the flexibility of this solutions employing CVaR as the risk mitigation procedure.

Also, a more stable solution is generally achieved at the expense of increasing the expected total costs. The resulting price of risk-aversion, though, varies significantly from one strategy for another, ranging from 4.35% for CVaR to less than 1% in the remaining approaches, on average. All in all, it seems that there is a typical trend of increasing production rates to avoid overly expensive lost sales solutions. This is particularly true for CVaR. However, whether high inventory levels are not an option, less conservative decision-makers should adopt the first-order stochastic dominance approach, thus avoiding the need of increasing substantially inventory levels to obtain a less risky-solution.

CVaR is especially relevant for minimizing the cost standard deviation, but the price of risk aversion is not necessarily low. Both FOSD and SD have presented attractive features, in the sense that both are capable of lowering the cost standard deviation reasonably for a rather low price. Certainly, less conservative managers would be even more willing to adopt SD and FOSD solutions over CVaR to ensure major risk reductions at a negligible price. However, FOSD is the most time-consuming strategy. Apparently, SOSD has provided less encouraging results for mitigating overall risks.

Interestingly, there is not a substantial difference in the risk indicators when we analyze the average values for each type of problem, i.e., independent or dependent demands; 10 or 20 products; 7 or 14 time periods; loose or normal capacity. However, most risk-averse models have achieved better risk reductions for dependent demand and normal capacity instances. There was no clear trend between the other characteristics of the instances and the risk mitigation strategies, suggesting that the qualitative behavior of the proposed approaches is rather similar regardless specific characteristics of the problems under study.

One can notice that the effect that CVaR has exhibited is more evident than that observed for SD, FOSD, and SOSD. This effect could possibly be further tuned in all strategies by setting different values for α (CVaR), benchmarks (FOSD and SOSD), and ϕ for both risk-averse models. A stronger effect, though, could hardly be obtained for SD, as the single control parameter (ϕ) that can be employed has been already considered at a high level (0.99). From a managerial perspective, finding effective benchmarks can be done for specific applications when real data is available and the decision-maker is able to include tighter thresholds for the overall costs, which may improve both FOSD and SOSD models and, as a consequence, stochastic dominance can be more competitive with mean-risk models, mainly if more efficient solution methods are available.

The idea of jointly reducing the variability of the expected costs and hedging against the occurrence of undesired (pessimistic) scenarios can be part of the strategy for enhancing the production planning in small soft-drink companies under demand uncertainty. In this respect, the results for the case-study helped us to identify potential effective risk management strategies to handle demand uncertainty/fluctuation for different risk attitudes. Whereas (CVaR) is especially endorsed by more conservative decision-makers who prefer implementing much less risky solutions, all the remaining solution approaches have the advantage of producing slightly less risky solutions at rather low prices. In this sense, stochastic dominance methods decidedly suit less conservative decision-makers.

5. Summary and Concluding Remarks

The results presented in this study concern the risk-averse two-stage production lot-sizing problem with stochastic demands. The idea of incorporating mathematical measures to reflect the attitude of the decision-makers towards risk is avoiding over costly solutions and controlling the dispersion of the recourse decisions in attempt to devise assertive production planning decisions. The importance of empirically showing how risk can be mitigated, and its corresponding tradeoffs, is motivated by the absence of a unrestrictedly recommended risk management approach, or a systematic comparisons amongst popular methods that evidence the benefits and drawbacks of the existing ones. Thus, we have explored four state-of-the-art risk-averse approaches to deal with production lot-sizing problems under stochastic demands within a risk management perspective. Semi-deviation and Conditional-Value-at-Risk measures were analyzed within a mean-risk framework that compromised expected costs and risk, whereas first- and second-order stochastic dominance constraints have enforced risk-aversion by directly satisfying target values or benchmarks. For the first time, the production lot-sizing problem was analyzed from the point of view of first- and second-order stochastic dominance constraints. In particular, the first-order stochastic dominance principle was modeled in a more tractable way based on the ideas presented in Luedtke (2008).

Our overall findings indicate that all analyzed risk-averse models are able to mitigate effectively the probability of experiencing undesirable cost outcomes. For this purpose, production rates are often increased, which not rarely implies in higher inventory levels. In particular, the numerical results prove that, from all analyzed models, CVaR is the one that reduces most dramatically the cost in worst scenarios. However, this result is achieved at expenses of a non-negligible increase of the expected cost. On the contrary, SD and FOSD have demonstrated numerically that they are suitable models for those decision-makers who desire to reduce risk at a low price. We would like also to highlight that our preliminary results indicate that the performance of the models based on stochastic dominance constraints is highly governed by the particular selection of the benchmark profile. For this reason, we are currently investigating a practical procedure for deriving benchmarks profiles to be used in decision-making problems including stochastic dominance constraints. Another interesting avenue of research is the development of a general approach to help decision-makers to select an adequate value of parameter ϕ in mean-risk models according to the their risk preferences. It is worth noting that all the results were supported by the case-study presented, which also confirmed the observed behaviour in a more specific setting related to a soft-drink company. From a factory standpoint, our findings bring to light ways to improve typical production planning decisions using risk management strategies that are able to provide more stable and accurate plans, avoiding unrealistically expenditures.

The problem we have studied is quite general. In practice settings, there might be several additional factors that could be considered in the model, such as scheduling decisions, sequence-dependent setups, among others, which inevitably increase its complexity. However, there is no empirical or theoretical evidence that leads us to believe that the behavior of the approaches would change completely amongst themselves even though other practical characteristics were included in the model, at least considering similar simple recourse structures for the recourse matrix.

Acknowledgements. We would like to thank Prof. Eli Angela Vitor Toso from Universidade Federal de São Carlos (Production Engineering Department) for sharing her views on how to construct an interesting research methodology in quantitative modeling. We thank both reviewers for carefully reading the manuscript and for their valuable comments and suggestions that have helped us to emphasize our contributions and overall methodology, which further improved the readability of the paper.

References

Alem, D., Clark, A., Moreno, A., 2016. Stochastic network models for logistics planning in disaster relief. European Journal of Operational Research 255 (1), 187 – 206.
URL http://www.sciencedirect.com/science/article/pii/S0377221716302788

Alem, D., Curcio, E., Amorim, P., Almada-Lobo, B., 2018. A computational study of the general lot-sizing and scheduling model under demand uncertainty via robust and stochastic approaches. Computers & Operations Research 90, 125 – 141.
URL http://www.sciencedirect.com/science/article/pii/S0305054817302289

Alem, D., Morabito, R., 2013. Risk-averse two-stage stochastic programs in furniture plants. OR Spectrum 35 (4), 773–806.
URL http://dx.doi.org/10.1007/s00291-012-0312-5

Alonso-Ayuso, A., Carvallo, F., Escudero, L. F., Guignard, M., Pi, J., Puranmalka, R., Weintraub, A., 2014. Medium range optimization of copper extraction planning under uncertainty in future copper prices. European Journal of Operational Research 233 (3), 711 – 726.
URL http://www.sciencedirect.com/science/article/pii/S0377221713007327

Aloulou, M. A., Dolgui, A., Kovalyov, M. Y., 2014. A bibliography of non-deterministic lot-sizing models. International Journal of Production Research 52 (8), 2293–2310.
URL http://dx.doi.org/10.1080/00207543.2013.855336

Amorim, P., Costa, A. M., Almada-Lobo, B., 2015. A hybrid path-relinking method for solving two-stage stochastic integer problems. International Transactions in Operational Research 22 (1), 113–127.
URL http://dx.doi.org/10.1111/itor.12084

Artzner, P., Delbaen, F., Eber, J.-M., Heath, D., 1999. Coherent measures of risk. Math. Finance 9, 203–228.

Barbaro, A., Bagajewicz, M. J., 2004. Managing financial risk in planning under uncertainty. AIChE Journal 50 (5), 963–989.

Broitman, D., Raviv, O., Ayalon, O., Kan, I., 2018. Designing an agricultural vegetative waste-management system under uncertain prices of treatment-technology output products. Waste Management 75, 37 – 43.
URL http://www.sciencedirect.com/science/article/pii/S0956053X1830062X

Buckley, J. J., 1986. Stochastic dominance: an approach to decision making under risk. Risk Analysis 6 (1), 35–41.

URL http://www.ncbi.nlm.nih.gov/pubmed/3602494

Dentcheva, D., Ruszczyński, A., 2009. Optimization with multivariate stochastic dominance constraints. Mathematical Programming 117 (1-2), 111–127.

Doostmohammadi, M., Akartunalı, K., 2018. Valid inequalities for two-period relaxations of big-bucket lot-sizing problems: Zero setup case. European Journal of Operational Research 267 (1), 86–95.

Eigel, M., Neumann, J., Schneider, R., Wolf, S., 2018. Risk averse stochastic structural topology optimization. Computer Methods in Applied Mechanics and Engineering 334, 470 – 482.
URL http://www.sciencedirect.com/science/article/pii/S0045782518300744

Escudero, L. F., Garín, M. A., Monge, J. F., Unzueta, A., 2018. On preparedness resource allocation planning for natural disaster relief under endogenous uncertainty with time-consistent risk-averse management. Computers Operations Research 98, 84 – 102.
URL http://www.sciencedirect.com/science/article/pii/S030505481830131X

Gollmer, R., Neise, F., Schultz, R., 2008. Stochastic programs with first-order dominance constraints induced by mixed-integer linear recourse. SIAM Journal on Optimization 19 (2), 552–571.

Graves, S., 2011. Uncertainty and production planning. In: Kempf, K., Keskinocak, P., Uzsoy, R. (Eds.), Planning production and inventories in the extended enterprise. International Series in Operations Research and Management Science. Vol. 151. New York: Springer, pp. 83–101.

Güngör, M., Ünal, A. T., Taşkın, Z. C., 2018. A parallel machine lot-sizing and scheduling problem with a secondary resource and cumulative demand. International Journal of Production Research 56 (9), 3344–3357.

Hu, Z., Hu, G., 2016. A two-stage stochastic programming model for lot-sizing and scheduling under uncertainty. International Journal of Production Economics 180, 198 – 207.
URL http://www.sciencedirect.com/science/article/pii/S0925527316301797

Huang, K., Küçükyavuz, S., 2008. On stochastic lot-sizing problems with random lead times. Operations Research Letters 36 (3), 303 – 308.
URL http://www.sciencedirect.com/science/article/pii/S0167637707001460

Karagul, H. F., Warsing Jr, D. P., Hodgson, T. J., Kapadia, M. S., Uzsoy, R., 2017. A comparison of mixed integer programming formulations of the capacitated lot-sizing problem. International Journal of Production Research, 1–21.

Koca, E., Yaman, H., Aktürk, M., 2018. Stochastic lot sizing problem with nervousness considerations. Computers & Operations Research 94, 23 – 37.
URL http://www.sciencedirect.com/science/article/pii/S0305054818300297

Krokhmal, P., Zabarankin, M., Uryasev, S., 2011. Modeling and optimization of risk. Surveys in Operations Research and Management Science 16 (2), 49–66.

Levy, H., 1992. Stochastic Dominance and Expected Utility: Survey and Analysis. Management Science 38 (4), 555–593.
URL http://pubsonline.informs.org/doi/abs/10.1287/mnsc.38.4.555

Li, H., Thorstenson, A., 2014. A multi-phase algorithm for a joint lot-sizing and pricing problem with stochastic demands. International Journal of Production Research 52 (8), 2345–2362.
URL http://dx.doi.org/10.1080/00207543.2013.864053

Luedtke, J., 2008. New formulations for optimization under stochastic dominance constraints. SIAM Journal on Optimization 19 (3), 1433–1450.
URL http://dx.doi.org/10.1137/070707956

Macedo, P. B., Alem, D., Santos, M., Junior, M. L., Moreno, A., Feb 2016. Hybrid manufacturing and remanufacturing lot-sizing problem with stochastic demand, return, and setup costs. The International Journal of Advanced Manufacturing Technology 82 (5), 1241–1257.
URL https://doi.org/10.1007/s00170-015-7445-z

Maes, J., McClain, J. O., van Wassenhove, L. N., 1991. Multilevel capacitated lotsizing complexity and lp-based heuristics. European Journal of Operational Research 53, 131–148.

Mahmutogullari, A. I., Cavus, O., Akturk, M. S., 2018. Bounds on risk-averse mixed-integer multi-stage stochastic programming problems with mean-cvar. European Journal of Operational Research 266 (2), 595 – 608.
URL http://www.sciencedirect.com/science/article/pii/S0377221717309517

Märkert, A., 2004. Deviation measures in stochastic programming with mixed-integer recourse. Ph.D. thesis, Universit at Duisburg-Essen.

Markowitz, H., 1952. Portfolio selection. The Journal of Finance 7(1), 77–91.
Markowitz, H., 1959. Portfolio selection: efficient diversification of investments. John Wiley & Sons.
Mulvey, J., Vanderbei, R., Zenios, S., 1995. Robust optimization of large-scale systems. Operations Research 43, 264–281.
Ogryczak, W., Ruszczyński, A., 1999. From stochastic dominance to mean-risk models: semideviations as risk measures. European Journal of Operational Research 116, 33–50.
Ogryczak, W., Ruszczyński, A., 2001. On consistency of stochastic dominance and mean-semideviation models. Mathematical Programming 89, 217–232.
Righetto, G. M., Morabito, R., Alem, D., 2019. Cash flow management by risk-neutral and risk-averse stochastic approaches. Journal of the Operational Research Society 0 (0), 1–14.
URL `https://doi.org/10.1080/01605682.2018.1524350`
Righi, M. B., 2017. A composition between risk and deviation measures. Annals of Operations Research, 1–15.
Riis, M., Schultz, R., 2003. Applying the minimum risk criterion in stochastic recourse programs. Computational Optimization and Applications 24, 267–287.
Rockafellar, R. T., Uryasev, S., 2000. Optimization of conditional value-at-risk. Journal of Risk 2, 21–41.
Rockafellar, R. T., Uryasev, S., 2002. Conditional value-at-risk for general loss distributions. Journal of Banking and Finance 26 (7), 1443–1471.
Ruszczynski, A., Shapiro, A., Jul. 2004. Optimization of risk measures. Risk and Insurance 0407002, EconWPA.
URL `http://ideas.repec.org/p/wpa/wuwpri/0407002.html`
Schultz, R., Tiedemann, S., 2006. Conditional value-at-risk in stochastic programs with mixed-integer recourse. Mathematical Programming 105, 365–386.
Shapiro, A., Dentcheva, D., , Ruszczynski, A., 2009. Lectures on stochastic programming: modeling and theory. SIAM, Philadelphia.
Takriti, S., Ahmed, S., 2004. On robust optimization of two-stage systems. Mathematical Programming Series A 99, 109–126.
Toso, E. A. V., Alem, D., 2014. Effective location models for sorting recyclables in public management. European Journal of Operational Research 234 (3), 839 – 860.
URL `http://www.sciencedirect.com/science/article/pii/S0377221713008588`
Trigeiro, W. W., Thomas, L. J., McClain, J. O., 1989. Capacitated lot sizing with setup times. Management Science 35 (3), 353–366.
Vladimirou, H., Zenios, S., 1997. Stochastic linear programs with restricted recourse. European Journal of Operational Research 101, 177–192.
Wu, T., Akartunali, K., Song, J., Shi, L., 2013. Mixed integer programming in production planning with backlogging and setup carryover: modeling and algorithms. Discrete Event Dynamic Systems 23 (2), 211–239.
URL `http://dx.doi.org/10.1007/s10626-012-0141-3`
Zhang, M., Küçükyavuz, S., Goel, S., 2014. A branch-and-cut method for dynamic decision making under joint chance constraints. Management Science 60 (5), 1317–1333.
Zhou, Z., Guan, Y., 2013. Two-stage stochastic lot-sizing problem under cost uncertainty. Annals of Operations Research 209 (1), 207–230.
URL `http://dx.doi.org/10.1007/s10479-013-1333-4`

Appendix A. Mathematical notation

We present a summarized list of the symbols used to formulate the mathematical models present in this paper. The symbols are listed in alphabetical order, with Latin characters presented first followed by Greek characters.

A.1 *Indices and Sets*

$k = 1, \ldots, K$	Products.
$p = 1, \ldots, \mathscr{P}$	Discrete benchmark points.
$t = 1, \ldots, T$	Time periods.
$\omega = 1, \ldots, \mathscr{W}$	Scenarios.

A.2 *Parameters*

c_k	Unit production cost of product k.
cap_t	Total capacity available in period t.
$\bar{d}_t$	Average (deterministic) demand in period t for all products.
d_{kt}	Deterministic demand for product k in period t.
$\tilde{d}_{kt}$	Stochastic demand for product k in period t.
$d_{kt\omega}$	Demand of product k in period t in scenario ω.
h_k	Holding cost per period for product k.
$M,\ M'$	Sufficiently large numbers.
p_k	Lost sale penalty cost for product k.
u_{kt}	Units of capacity required to manufacture product k in period t.
α	Confidence level in the definition of CVaR.
β_p	Cumulative probability of benchmark point p.
ε_p	Expected cost target of benchmark point p.
π_ω	Probability associated with scenario ω.
ρ_p	Target value (cost threshold) of benchmark point p.
ϕ	Weight ($\phi \in [0,1]$) trading of mean and risk terms in mean-risk models.

A.3 *Decision Variables*

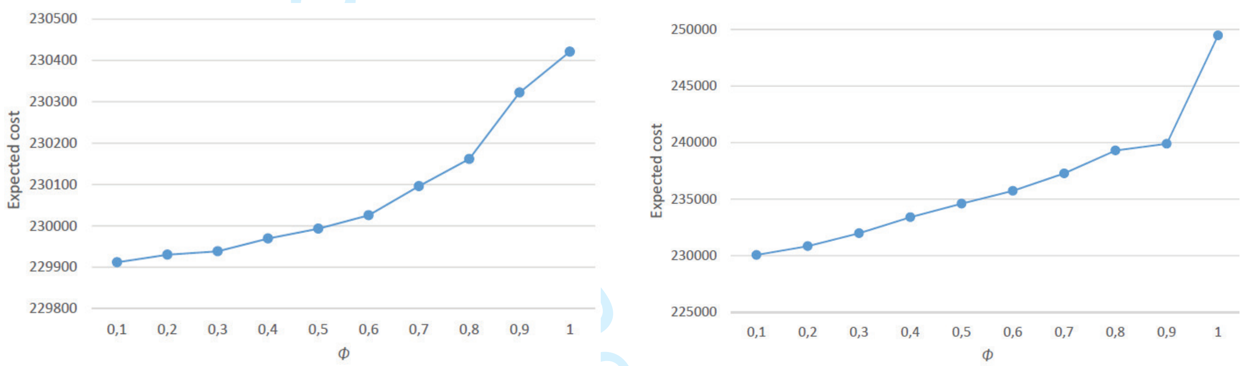
$l_{kt\omega}$	Total of lost sales of product k in period t for scenario ω.
$s_{kt\omega}$	Inventory level of product k in period t for scenario ω.
x_{kt}	total Production of product k in period t.
$\gamma_{\omega p}$	Auxiliary variable; captures the amount by which the cost of scenario ω exceeds the threshold ρ_p of benchmark point p in scenario ω in SOSD.
δ_ω	Auxiliary variable; captures the maximum between the expected cost and the cost of scenario ω in SD.
η	Value-at-Risk (VaR) threshold in the definition of CVaR.
δ_p	Auxiliary variable; slack variable for benchmark point p probability in FOSD-O and FOSD.
δ'_p	Auxiliary variable; slack variable for benchmark point p expected cost shortfall in SOSD.
$\theta_{\omega,p}$	Auxiliary variable; indicates if the cost of scenario ω exceeds the threshold ρ_p of benchmark point p in FOSD-O and FOSD.
ϑ_ω	Auxiliary variable; captures a cost deviation greater than VaR in scenario ω.
λ_ω	Auxiliary variable; captures the cost of scenario ω in SD.
φ_p	Auxiliary variable; accumulates the probability of scenarios exceeding the threshold ρ_p of benchmark point p in FOSD.

Appendix B. Sensitivity to parameter ϕ

This appendix shows the sensitivity of the obtained results relative to parameter ϕ used in the mean-risk models analyzed in this paper (SD and CVaR). Note that parameter ϕ is a weighting factor that is used to model the tradeoff between expected cost and risk, which depends on the preferences on the decision-maker. A conservative decision-maker focusing on minimizing the risk, would choose a value of ϕ close to 1 to increase the weight of the risk-measure in the objective functions (7) and (14). On the other hand, another decision-maker might be willing to assume a higher risk in the hope of obtaining a lower cost, so that its selected value for ϕ would be close to 0.

In order to quantify the effect of ϕ on the obtained results, instance `1-p10-t7-c0.85-ID` has been solved considering 10 values of ϕ ranging between 0.1 and 1. We would like to emphasize that it has been observed that the impact of ϕ on the results of this particular instance is qualitatively equivalent to those obtained in the rest of instances considered in this paper.

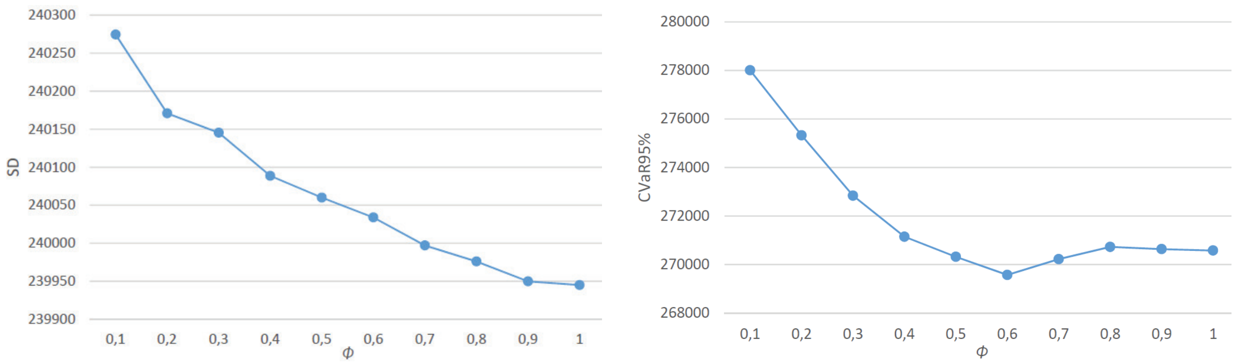
Figure B1 depicts the resulting expected costs using the semideviation (Figure B1a) and the CVaR at 95% (Figure B1b) as risk metrics for different values of ϕ. As expected, expected costs grow as the decision maker becomes more conservative (larger values of ϕ). It is observed that the expected costs in SD and CVaR increase 0.2% and 8.7%, respectively, when ϕ grows from 0.1 to 1. These results indicate that the expected costs in CVaR are much more sensitive to the value of ϕ than those in SD.



(a) SD (b) CVaR

Figure B1.: Effect of ϕ on the expected cost.

Figure B2 represents the value of the risk metric, the cost semideviation in SD (Figure B2a) and the $\mathsf{CVaR}_{0.95}$ in CVaR (Figure B2b), for different values of ϕ. Opposite to the expected cost, the quantification of risk decreases as ϕ grows. Again, we observe that, in risk terms, CVaR is much more sensitive to the value of ϕ than SD. In particular we observe that in the interval $\phi \in [0.1, 1]$, the semideviation of the cost decreases 0.1% in SD, whereas the $\mathsf{CVaR}_{0.95}$ is reduced 2.7% in the CVaR approach. These results confirm that, tunning the value of parameter ϕ, CVaR is able to carry out a risk management in a much more effective manner than SD.



(a) SD (b) CVaR

Figure B2.: Effect of ϕ on the risk measure.

Appendix C. Results for the Case-Study

Table C1 provides the results obtained from the soft-drink company case study for all 10 instances. In general, the results provided for each instance are qualitatively similar to those observed in average. The most striking case happens in instance 1, which has associated the one of the smallest expected costs. In this instance, CVaR decreases significantly the standard deviation of the cost, 35.884%. However, the cost associated with the worst-case scenario is marginally increased 0.234%, being this the only instance in which this is observed. This amount is much worse than the average reduction of 9.57%. In the same manner, FOSD and SOSD are also unable to efficiently decrease the worst-case cost in this instance, but SD shows a small reduction.

Table C1.: Summary of results for the larger instances inspired by the case-study in terms of expected value ($\mathbb{E}$xpec), standard deviation (StDev), worst-case scenario (Wc), and $\mathsf{CVaR}_{0.95}$.

Instance	Expec					StDev				
	Benchmark	SD (%)	CVaR (%)	FOSD (%)	SOSD (%)	Benchmark	SD (%)	CVaR (%)	FOSD (%)	SOSD (%)
1	287,164,822	0.207	7.419	0.097	0.033	41,907,403	− 7.669	− 35.884	− 3.400	− 1.456
2	294,034,124	0.456	6.997	0.205	0.018	37,651,624	− 18.525	− 52.070	− 7.771	− 2.112
3	287,875,623	0.353	8.289	0.230	0.057	32,012,487	− 11.864	− 40.080	− 4.608	− 2.815
4	288,547,018	0.083	5.069	0.109	0.060	30,130,224	− 12.909	− 41.520	− 9.858	− 2.119
5	311,099,540	0.562	9.877	0.058	0.006	44,604,237	− 15.679	− 48.671	− 5.939	− 1.975
6	292,807,821	0.207	7.308	0.372	0.033	27,092,785	− 11.199	− 38.029	− 7.225	− 2.133
7	299,829,859	0.331	7.146	0.172	0.050	33,543,944	− 14.547	− 47.613	− 6.947	− 3.264
8	297,664,985	0.352	7.957	0.162	0.020	34,718,220	− 13.728	− 43.291	− 4.612	− 3.343
9	298,922,726	0.530	11.070	0.198	0.055	34,807,775	− 11.998	− 46.960	− 5.315	− 2.927
10	286,290,668	0.359	10.416	0.300	0.079	33,528,143	− 11.219	− 52.111	− 6.896	− 4.834
Average	294,423,718	0.344	8.155	0.190	0.041	34,999,684	− 12.934	− 44.623	− 6.257	− 2.698

Instance	Wc					$\mathsf{CVaR}_{0.95}$				
	Benchmark	SD (%)	CVaR (%)	FOSD (%)	SOSD (%)	Benchmark	SD (%)	CVaR (%)	FOSD (%)	SOSD (%)
1	457,396,294	− 0.508	0.234	0.531	0.201	413,225,095	− 2.12	− 6.37	− 1.25	− 0.53
2	405,909,516	− 5.094	− 11.702	− 1.505	− 0.600	390,453,070	− 4.19	− 9.57	− 1.51	− 0.60
3	413,387,682	0.137	− 1.126	2.187	0.333	386,153,661	− 2.61	− 5.83	− 1.42	− 0.68
4	428,554,298	− 2.698	− 14.255	− 1.701	− 0.777	369,788,307	− 3.07	− 8.08	− 2.62	− 0.86
5	511,540,786	0.861	− 6.573	0.149	0.252	446,596,569	− 3.71	− 9.21	− 1.66	− 0.50
6	389,643,660	− 0.686	− 6.454	− 2.076	− 0.610	364,109,929	− 1.56	− 3.49	− 0.75	− 0.26
7	436,492,442	− 3.795	− 16.489	− 1.228	− 1.107	400,632,638	− 2.84	− 10.65	− 1.57	− 0.49
8	438,365,554	− 2.049	− 11.548	0.928	− 0.150	395,163,327	− 2.53	− 7.89	− 0.85	− 0.73
9	422,976,526	1.114	− 12.888	0.044	− 0.463	409,486,599	− 2.14	− 10.80	− 1.09	− 0.69
10	440,087,733	− 1.159	− 14.899	0.518	− 1.410	387,695,429	− 1.07	− 10.00	− 0.19	− 1.19
Average	434,435,449	− 1.388	− 9.570	− 0.215	− 0.433	396,330,462	− 2.584	− 8.190	− 1.291	− 0.653